Math 153
Spring 2016
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SOLUTIONS TO EXERCISES FROM `math153exercises12b.pdf`

All of the exercises considered here are *Additional exercises* which are in the cited exercise file. Solutions to other exercises related to Unit 12 are given in the files `math153solutions14.pdf` and `math153solutions14a.pdf`.

4. (a) The auxiliary polynomial equation for this difference equation is $r^2 - 6r + 8 = 0$ and its roots are $r = 2, 4$; therefore the general solution has the form $2^n P + 4^n Q$. By assumption the initial values are $3 = x_0 = P + Q$ and $2 = x_1 = 2P + 4Q$. If we solve these equations we obtain $P = 5$ and $Q = -2$, and therefore the solution is given by $5 \cdot 2^n - 2 \cdot 4^n$.$\blacksquare$

(b) The auxiliary polynomial equation for this difference equation is $r^2 - 5r + 4 = 0$ and its roots are $r = 1, 5$; therefore the general solution has the form $P + 5^n Q$. By assumption the initial values are $0 = x_0 = P + Q$ and $6 = x_1 = P + 5Q$. If we solve these equations we obtain $P = -\frac{3}{2}$ and $Q = \frac{3}{2}$, and therefore the solution is given by $\frac{3}{2} \cdot 5^n - \frac{3}{2}$.$\blacksquare$

(c) The auxiliary polynomial equation for this difference equation is $r^2 + 5r + 6 = 0$ and its roots are $r = -2, -3$; therefore the general solution has the form $(-2)^n P + (-3)^n Q$. By assumption the initial values are $0 = x_0 = P + Q$ and $1 = x_1 = -(2P + 3Q)$. If we solve these equations we obtain $P = 1$ and $Q = -1$, and therefore the solution is given by $(-2)^n - (-3)^n$.$\blacksquare$

(d) The auxiliary polynomial equation for this difference equation is $r^2 - 3r + 2 = 0$ and its roots are $r = 1, 2$; therefore the general solution has the form $P + 2^n Q$. By assumption the initial values are $1 = x_0 = P + Q$ and $2 = x_1 = P + 2Q$. If we solve these equations we obtain $P = 0$ and $Q = 1$, and therefore the solution is given by 2^n.$\blacksquare$

(e) The auxiliary polynomial equation for this difference equation is $r^2 - 9 = 0$ and its roots are $r = \pm 3$; therefore the general solution has the form $3^n P + (-3)^n Q$. By assumption the initial values are $2 = x_0 = P + Q$ and $-1 = x_1 = 3P - 3Q$. If we solve these equations we obtain $P = \frac{5}{6}$ and $Q = \frac{7}{6}$, and therefore the solution is given by $\frac{5}{6} \cdot 3^n + \frac{7}{6} \cdot (-3)^n$.$\blacksquare$

(f) The auxiliary polynomial equation for this difference equation is $3r + 2 = 0$ and its root is $r = -\frac{2}{3}$; therefore the general solution has the form $(-\frac{2}{3})^n K$. By assumption the initial value is $4 = x_0 = K$. Therefore the solution is given by $4 \cdot (-\frac{2}{3})^n$.$\blacksquare$

(g) The auxiliary polynomial equation for this difference equation is $0 = r^3 - 6r^2 + 11r - 6$, and the right hand side factors into the product $(r-1)(r-2)(r-3)$, so the roots are $r = 1, 2, 3$. Therefore the general solution has the form $A + 2^n B + 3^n C$. By assumption the initial values are $0 = x_0 = A + B + C$, $1 = x_1 = A + 2B + 3C$ and $1 = x_2 = A + 4B + 9C$. If we solve these equations we obtain $A = -2$, $B = 3$ and $C = -1$, and therefore the solution is given by $(-2) + 3 \cdot 2^n - 3^n$.$\blacksquare$