Additional exercises for Assignment 3

1. Suppose that we are given a proper trapezoid **ABCD**, by which we mean that **AB** is parallel to **CD** but **AD** is not parallel to **BC**. Is it possible for $\angle$ **A** and $\angle$ **D** to have the same measure? What if we replace $\angle$ **D** by $\angle$ **C** or $\angle$ **B?** For all cases in which the answer is yes, sketch a convincing example, and for all cases in which the answer is no give a convincing reason why equality is impossible. You need not give a detailed proof, however.

2. Burton mentions that one has Side-Angle-Side, Angle-Side-Angle, Side-Side-Side and Angle-Angle-Side congruence theorems for triangles. One easy way to construct a counterexample to Side-Side-Angle is to start with an equilateral triangle **ABC**, take a point **D** such that **C** is the midpoint of the segment **BD**, and consider the correspondence Δ **ADC** $\leftrightarrow$ Δ **ADB**. Find the measures of $\angle$ **ADC**, $\angle$ **ADB**, $\angle$ **CAD** and $\angle$ **BAD. [*<u>Hint</u>* :** Why is Δ **ACD** isosceles**?]**

3. Given three lengths a, b, c and three angle measures x, y, z several results in geometry show that these numbers must satisfy certain conditions. Specifically, assume that we are given Δ **ABC** such that the sides opposite vertices **A, B, C** have lengths a, b, c and the vertex angles at **A, B, C** have measures x, y, z respectively. For example, we have [1] the sum of any two lengths is greater than the third, [2] b = c if and only if z = y and similarly if the roles of the variables are interchanged, [3] x + y + z = 180, [4] $c^2 < a^2 + b^2$ if and only if z < 90. Determine which of these reasons imply that one cannot construct a triangle whose measures are partially given as follows (in some cases more than one reason might be needed):

(a) a = 8, b = c = 6, y = z = 60.
(b) a = 6, b = 7, c = 9, z = 93.
(c) a = 1, b = 2, c = 3.

4. Given Δ **ABC** one can find several particularly interesting points associated to it. If one joints each vertex to the midpoint of the opposite side, then it turns out that these three lines all pass through a common point known as the **centroid** of the triangle. Similarly, if one drops perpendiculars from each vertex to the opposite side, then these three lines all pass through a common point known as the **orthocenter** of the triangle. Finally, there is a unique circle that contains all three vertices and its center is called the **circumcenter** of the triangle. In each of the examples below, determine whether each of these three points lies inside the triangle, on one of the vertices, on one of the sides between two vertices, or outside the triangle.

(a) An equilateral triangle.
(b) An isosceles right triangle.
(c) A 120 – 30 – 30 isosceles triangle.