0. Introduction and general information

(Burton, ***Preface***)

We shall begin with some brief remarks on the motivation for this course and its objectives. There are many valid intellectual reasons for studying the history of mathematics, but probably the most compelling reason for this particular course is the State of California's History of Mathematics requirement for teacher certification in mathematics. The latter appear in the 2003 document on standards for teacher preparation, and the portions on the history of mathematics stress major developments whose impacts on middle and high school mathematics have been particularly strong. We shall discuss the latter specifically when we summarize the course contents later in this unit.

For several reasons it is not feasible to cover the entire history of mathematics in a single ten week course at the undergraduate level. One way of seeing this is to look at the lengths of some standard books on the subject. Morris Kline's ***Mathematical Thought from Ancient to Modern Times*** has 51 chapters and is over 1250 pages long. The second edition of C. B. Boyer and U. C. Merzbach's ***History of Mathematics*** is nearly 750 pages long. The much older two volume ***History of Mathematics*** by D. E. Smith from the mid 1920s has a combined length of about 1350 pages. The official text for this course, the fifth edition of D. M. Burton's ***History of Mathematics***, is also about 750 pages long and the density of words per page is higher than most if not all of the other books mentioned above. Going through any of these books completely would mean covering at least 75 pages of text per week. Beyond simple page counting, there are several compelling reasons why it is unreasonable to expect that any account of mathematical history to say much if anything about mathematical developments during the past 70 years without discussing mathematical material that is generally not discussed below the graduate level.

Therefore it is necessary to say something about the sorts of topics that we shall cover and how we shall do so. First of all, it makes sense to focus attention on the historical development of material that undergraduates have seen in the courses taken during the first two years of college. Second, due to time constraints our main emphasis will be on the evolution of mathematics as it currently exists in our contemporary civilization. All cultures have had some form of mathematics or pre-mathematics. In particular, many societies have independently developed similar ideas, often from extremely different perspectives, and it must be stressed that the limited discussion of such achievements reflects our time constraints and by no means signifies any lack of appreciation or respect. Third, since this is an upper division mathematics course it will contain a very significant amount of mathematics, including worked out problems in class, homework exercises, and even some proofs. Finally, our perspective for this course will be that the development of mathematics is about people learning how to solve problems and trying to understand natural phenomena.

Some general objectives

One reason for studying history is to increase a person's consciousness and awareness of his or her world. In this course, one particular goal is to reinforce a prospective teacher's grasp of the material that is taught in middle and high schools, including calculus. It is hoped that such an enhanced command of mathematics will be helpful for teaching the standard classes as well as other educational purposes; for example, communicating mathematics outside the classroom and continued professional growth after certification.

Nearly every discussion of reasons for studying history ultimately includes some version of a widely known quotation from the American philosopher George Santayana:

Those who do not learn from the past are doomed to repeat it.

This applies differently to individual subjects, so some remarks on its meaning for mathematics seem appropriate. Because of the topics it covers, mathematics as a whole is able to learn from its past mistakes relatively well. However, an individual today must still face many of the same learning issues that past societies had to address. Two issues that come to mind immediately are understanding the difference between approximate and exact answers and finding a way to organize the subject adequately. Some understanding of the history of mathematics can be helpful for such educational purposes.

History as a subject is something that some people think is absolutely fascinating, others think is interesting or worthwhile, and still others think is hopelessly boring or even utterly useless. The first of these is largely a matter of personal taste and as such cannot be taught. However, one goal of this course is to address the second and third views. We have already noted that emphasis will be on points that are relevant to teaching mathematics above the first few grades of elementary school and useful for a teacher to know. Also, there will be considerable emphasis on general issues concerning the evolution of mathematics as we know it as opposed to a relatively dry recitation of people, places and events in chronological order.

Topics to be covered in the course

We begin by paraphrasing the present California requirements for teacher preparation in mathematics (*Reference:* Mathematics Preparation in California: Standards of Quality and Effectiveness for Subject Matter Programs. Mathematics Subject Matter Panel, 2001–2003. California Commission on Teacher Credentialing, Sacramento CA, 2003). This document is available online at the following address:

http://www.ctc.ca.gov/educator–prep/standards/SSMP–Handbook–Math.pdf

Four major developments are cited on page 38 of this document (= online page 46). These are the development of numeration systems, the gradual evolution of algebraic symbolism, the ultimate emergence of differential and integral calculus from earlier ideas, with some dating back to classical Greek mathematics, and the changing role of logic and proof that has accompanied the growth of mathematics. More details on our coverage of these developments appears at the end of this unit.

The standards in the State document also include the impacts of mathematical advances on human society and thought, the cultural connections of the subject, and its impact on society. The following quotation from the Preface to Morris Klline's book ***Mathematics in Western Culture*** provides a good summary of goals in these directions:

> Mathematics has become a major cultural force in Western civilization. Almost everyone knows that mathematics serves the very practical purpose of dictating engineering design. Fewer people seem to be aware that mathematics carries the main burden of scientific reasoning and is the core of the major theories of physical science. It is even less widely known that mathematics has determined the direction and content of much philosophic thought, has destroyed and rebuilt religious doctrines, has supplied substance to economic and political theories, has fashioned major painting, musical, architectural, and literary styles, has fathered our logic, and has furnished the best answers we have to fundamental questions about the nature of man and his universe. As the embodiment and most powerful advocate of the rational spirit, mathematics has invaded domains ruled by authority, custom, and habit, and supplanted them as the arbiter of thought and action. Finally, as an incomparably fine human achievement mathematics offers satisfactions and aesthetic values at least equal to those offered by any other branch of our culture

Having mentioned two of Kline's works already, we should note that Kline's views on twentieth century mathematics (by implication, twenty-first century mathematics as well!) were extremely controversial, and although they are often presented quite attractively and forcefully, a closer examination of the facts often indicates serious difficulties with many crucial points. However, such controversial opinions are rarely stated explicitly or disruptively in his historical writings, and this applies particularly to the portions about the development of the subject through the end of the nineteenth century. More will be said about this towards the end of the course.

When one examines the history of any scientific subject, it is inevitable that at least some outside historical developments must be recognized. However, when doing so it is very tempting to get into topics that are extremely controversial or beyond one's professional expertise or at best remotely tied to the development of mathematics itself. Since there is already more than enough mathematically related material to discuss in this course, it will contain far less commentary on the historical developments outside of mathematics than one finds in many textbooks (for example, in some portions of Burton).

We shall also place more emphasis on mathematical developments themselves as opposed to the lives of the many talented individuals from all over the world who have made significant contributions to the subject. The excellent online ***MacTutor History of Mathematics Archive*** located at the site

http://www–groups.dcs.st–and.ac.uk/~history/index.html

contains extensive biographical information for more than 1100 mathematicians (including many women and individuals from non-Western cultures) as well as an enormous amount of other material related to the history of mathematics.

Here is a summary of the specific topics to be covered in the course. The mathematical legacy of ancient civilizations before the Greeks will be covered briefly, with most of the attention devoted to the legacies of the Egyptian and Mesopotamian civilizations. There will be more extensive coverage of the mathematical legacy of Greek civilization over the roughly thousand year period from around 600 B. C. E. through the fourth century of the present era. Particular emphasis will be placed on the period between 400 B. C. E. and

200 B. C. E. when two of the best known contributors, Euclid and Archimedes, were active. Between the twilight of Greek mathematics and the renewed mathematical activity in Europe around the beginning of the thirteenth century, there was a transitional period in which Islamic civilizations preserved many achievements of ancient Greek mathematics and combined them with contributions ideas from Eastern civilizations, and particularly from Hindu mathematicians; the revival of European mathematics during the late Middle Ages relied very heavily on the mathematical legacy of Islamic culture. The initial surge of activity in the thirteenth century will be covered briefly, with more attention to the major progress and innovations during the sixteenth century that went well beyond all previous achievements. By the year 1600, mathematics was poised to advance even more rapidly, and it has done so at an ever increasing rate since that time. Not surprisingly, there will be fairly extensive discussion of related to the subjects now described by the term precalculus (trigonometry had already been worked out fairly well, but logarithms and coordinate geometry really belong to the seventeenth century), and the strong ties between mathematics advances in and physics during the seventeenth century will also be discussed. However, the central topic of interest from this century is the development of calculus itself. Earlier work anticipating calculus before Newton and Leibniz will be covered briefly, with more attention on the specific contributions of Newton and Leibniz. The mathematical developments of the eighteenth century include many enhancements to the work of Newton and Leibniz as well as renewed attention to many earlier areas, and the discussion of eighteenth century mathematics in this course will concentrate on such themes. However, as one moves into the eighteenth and even later centuries, progress in mathematics becomes increasingly remote from topics studied in standard precollege and lower level undergraduate classes. Therefore the treatment of later mathematical history will be confined to general trends and developments affecting the material in such mathematics courses. Since this comes at the end of the course, it is probably not surprising that the discussion will be fairly brief. As noted before, our primary interest in this period will involve areas of mathematics treated through the standard lower division undergraduate courses.

Perhaps the most unfortunate omission from the course will be the development of probability theory. This is an important topic in the history of mathematics, but currently most classes below the college level only mention some aspects of probability theory in passing and concentrate on more basic topics instead. The State standards for mathematics curricula through high school contain some extremely ambitious goals, and if these become widely implemented then the content of history of mathematics courses might require corresponding revisions. For the record, the document on curriculum standards is available online at the following address.

http://www.cde.ca.gov/re/pn/fd/documents/math–stnd.pdf

There are sections discussing topics from probability and statistics in the portions dealing with Grade 6, Algebra II, and both regular high school and AP courses on Probability and Statistics itself.

Some further references

We have already included a few references to other books and online sites for further information, and we shall conclude this unit by listing a few more. There is no way that a list of this sort can include everything that is worthy of inclusion; the web sites themselves contain numerous links to other sites, and looking up the sales information for a specific book on www.amazon.com or www.bn.com will usually provide names of similar or related books. There are several widely used textbooks on the history of mathematics, and each has its own strengths. Reading accounts of topics from at least one book besides the course text is often very useful, and doing so is seriously recommended.

HISTORY OF MATHEMATICS SITES

http://www–groups.dcs.st–and.ac.uk/~history/index.html

http://aleph0.clarku.edu/~djoyce/mathhist/mathhist.html

http://dir.yahoo.com/science/mathematics/history/

http://en.wikipedia.org/wiki/History_of_mathematics

http://www.dean.usma.edu/math/people/rickey/hm/mini/textbooks.html

MATHEMATICS ENCYCLOPEDIA SITES

http://mathworld.wolfram.com

http://en.wikipedia.org/wiki/Main_Page

http://planetmath.org

HISTORY OF MATHEMATICS TEXTS

C. B. Boyer, ***A history of mathematics.*** Revised reprinting of the second edition, with a foreword by Isaac Asimov. Revised and with a preface by U. C. Merzbach. *John Wiley & Sons, Inc., New York,* 1991. ISBN: 0–471–54397–7

R. Calinger, J. Brown and T. West, ***A contextual history of mathematics.*** *Prentice–Hall, Saddle River, NJ,* 1999. ISBN: 0–023–18285–7.

R. Cooke, ***The history of mathematics. A brief course.*** *John Wiley & Sons, Inc., New York,* 1997. ISBN: 0–471–18082–3.

H. Eves, ***An introduction to the history of mathematics.*** Sixth edition. *Saunders College Publishing, Philadelphia, PA,* 1990. ISBN: 0–03–029558–0.

V. Katz, ***A history of mathematics. An introduction.*** Second Edition. *Addison–Wesley, Boston, MA,* 1998. ISBN: 0–321–01618–1.