NAME: _____

Mathematics 153, Spring 2005, Examination 1

Point values are indicated in brackets.

1. [*15 points*] The sum of unit fractions

$$r = \frac{1}{7} + \frac{1}{42}$$

can also be written in the form

$$\frac{1}{n} + \frac{1}{2n}$$

for some value of n. Find the two unit fractions in this expression for r.

SOLUTION.

We need to solve the equation

$$\frac{1}{n} + \frac{1}{2n} = \frac{1}{7} + \frac{1}{42}$$

and if we simplify both sides of the equation this redues to

$$\frac{3}{2n} = \frac{1}{6}\,.$$

This equation is equivalent to $2n = 18$, which means $x = 9$. To complete the problem we have to substitute this back into the original equation to show that

$$\frac{1}{18} + \frac{1}{9} = \frac{1}{6}\,.\blacksquare$$

2. [*25 points*] Given an odd prime p prove directly that $6\,p$ is never a perfect number.

As noted in class, it will suffice to do this when $p \neq 3$.

SOLUTION.

The proper divisors of p are 1, 2, 3, 6, p, $2p$ and $3p$. These add up to $6p + 12$, which is greater than $6p$, and therefore $6p$ is not a perfect number.∎

3. [*15 points*] Find a positive integer b such that $7^2 + b^2 = (b+1)^2$. (What is $(b+1)^2 - b^2$?)

SOLUTION.

The key to doing this is to note that $b^2 + 2b + 1 = (b+1)^2$. In order to solve this equation we need to find the value of b such that $2b + 1 = 49 = 7^2$. Solving this yields $b = 24$, and therefore we have the equation $24^2 + 7^2 = 25^2$.■

4. [*20 points*] For each of the following mathematicians, identify the time during which they worked using the following key: A = before 700 B.C.E., B = between 700 B.C.E. and 500 B.C.E., C = between 500 B.C.E. and 400 B.C.E., D = between 400 B.C.E. and the death of Alexander the Great in 323 B.C.E., E = between the death of Alexander the Great and 200 B.C.E., F = after 200 B.C.E. A given letter may be the correct answer more than once.

Euclid ANSWER: E

Eudoxus ANSWER: D

Proclus ANSWER: F

Thales ANSWER: B

Zeno ANSWER: C

5. [*25 points*] For each of the following mathematical developments, identify when they took place using the following key: A = before 700 B.C.E., B = between 700 B.C.E. and 500 B.C.E., C = between 500 B.C.E. and 400 B.C.E., D = between 400 B.C.E. and the death of Alexander the Great in 323 B.C.E., E = between the death of Alexander the Great and 200 B.C.E., F = after 200 B.C.E. A given letter may be the correct answer more than once.

Condition of Eudoxus used to work with irrational numbers. ANSWER: D

Deductive proofs introduced into mathematics. ANSWER: B

Extensive results on Pythagorean triples known. ANSWER: A

Hippocrates' results on areas of lunes (crescent shapes). ANSWER: C

Organization of basic Greek mathematics in the *Elements*. ANSWER: E

Quadratrix of Hippias first used to solve construction problems. ANSWER: C