Math 153
Spring 2005
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UPDATED GENERAL INFORMATION — MARCH 31, 2005

Here is the first homework assignment, which is due in class on **Wednesday, April 6, 2005.** "Burton" refers to the course text by Burton.

- Burton, p. 26: 3, 4, 5
- Express the ordinary fractions

$$\frac{2}{9}\ ,\ \frac{1}{25}\ ,\ \frac{1}{100}\ ,\ \frac{1}{125}$$

  in sexagesimal form.
- The number 1;24,51,10 appears on a Babylonian cuneiform tablet. Express it on more familiar terms.
- Prove the assertion in the notes that a rational number r satisfying $0 < r < 1$ has only finitely many Egyptian fraction expansions with a fixed length $L > 1$. [*Hint:* Proceed by induction on the length. What happens if $L = 1$? Suppose that the result is known for length L and proceed to length $L + 1$. If one has an Egyptian fraction expansion of lemgth $L + 1$, why must one of the summands be greater than $r/(L + 1)$? Show this implies that the denominator of at least one summand is $\leq (L + 1)/r$. For each positive integer $m < (L + 1)/r$ why do the induction hypothesis and the condition $0 < \frac{1}{m} < r$ imply that the fraction $r - \frac{1}{m}$ has only finitely many Egyptian fraction expansions of length L? How can one conclude the proof using this information?]
- Burton, p. 48: 8, 13
- Express $\frac{p}{11}$ as an Egyptian fraction for each p such that $2 \leq p \leq 10$.
- Burton, p. 58: 9
- Burton, p, 67: 4, 6, 13abc
- Exercise 5(b) on page 75 of Burton mentions an incorrect Bablyonian formula for the area of an isosceles trapezoid $ABCD$:

$$\text{area} \quad = \quad \frac{(a+c)\cdot(b+d)}{4}$$

  Here a and c are the lengths of the two parallel sides and b and d are the lengths of the nonparallel sides. The files `trapezoidABCD.*` — where $* =$ `ps`, `pdf` or `jpg` — give an illustration that is consisten with Exercise 9 on page 58 of Burton. Using the formula in the exercise on page 58, find the actual area of an isosceles trapezoid in term of the lengths of the sides and trigonometric functions of the angle θ at the vertex A. Recall that the nonparallel sides have equal length, the measures of the vertex angles at A and B are equal, and the measures of the vertex angles at C and D are supplementary to those at A and B (recall that two angles are supplementary if their measures add up to 180°). What is the ratio of the actual area to the figure given by the formula if the vertex angles at A and B are 60° angles?