Math 153
Spring 2005
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UPDATED GENERAL INFORMATION — MAY 13, 2005

Here is things to think about for the second midterm examination, which will take place on **Wednesday, May 18, 2005.** As noted in class, 65 per cent will be problems and 35 per cent will cover history, with more emphasis this time on matching the names of mathematicians to their known works.

On the mathematical problems side, here are some suggestions.

(1) Understand the unique factorization property for integers, the concept of greated common divisor d of two integers a and b, and the relationship between the latter and expressions of the form $pa + qb$ where p and q are integers.

(2) Know how to work with simple Diophantine equations and find integer solutions, particularly in the linear case. In particular, know when integer equations like $ax + by = c$ and $x^2 = ay + b$ can be solved and what the solutions are. For the latter, the following problem is worth considering. The equation $x^2 = 7y + r$ for $r = 1, 2, 3, 4, 5, 6$ can be solved for three values of r and cannot be solved for the other three values. Find the values for which it can be solved and for which it cannot be solved. Worth considering but less important is to see if you can describe all solutions in the cases where the equation can be solved (*Hint:* In each case the solutions have the form $7z \pm s$ for some value of s.

(3) Know Pappus' Centroid Theorem(s) and how to use them to computed surface areas, volumes or locations of centroids. Also know how to use such results to obtain Archimedean-type volume formulas comparing the volumes of two closely related figures. Examples involving spheres and ellipses were given in the homework, and in class we noted that one can use the formulas to compute the volume area of the doughnut shaped solid obtained by rotating a circle like $x^2 + (y - b)^2 = b^2 + c^2$ about the x-axis and also the area of the surface bounding this solid region. Here is an example of an Archimedean-type problem: Suppose we have a cone inscribed inside an equilateral triangle, and suppose further that it is situated so that the axis joining one vertex to the midpoint of the opposite side is the x-axis. The center of the triangle will then be $2/3$ of the way between the vertex and the midpoint of the opposite side, and this will be the radius of the inscribed circle. Find the ratio of the volumes of the corresponding solids of revolution (a sphere which is inscribed in a cone). By the Pythagorean Theorem, if b is the length of a side of the original triangle, then an altitude joining a vertex to the midpoint of an opposite side will have length $\frac{1}{2}b\sqrt{3}$.

(4) Greek mathematicians and astronomers did trigonometry using the chord function $\mathrm{crd}\,\theta$ mentioned in the notes. Its value is given by considering an isosceles triangle whose legs have length 1 and the angle determined by the legs has measure θ. You should be able to express $\mathrm{crd}\,\theta$ in terms of standard trigonometric functions (particularly $\sin(\theta/2)$, to give a formula for $\mathrm{crd}\,\frac{1}{2}\theta$ in terms $\mathrm{crd}\,\theta$ and to prove the identity

$$\mathrm{crd}^2\,\theta \;+\; \mathrm{crd}^2\,(180^\circ - \theta) \;=\; 4.$$

The historical review appears on the next page.

Historical summary

(380 BCE - 320 BCE) Menaechmus — Early work on conics, duplication of cube using intersecting parabolas.

(325 BCE - 265 BCE) Euclid — Organized fundamental mathmatial material in the *Elements*, including material on geometry, number theory and irrational quantities.

(287 BCE - 212 BCE) Archimedes — Computations of areas and volumes, study of spiral curve, methods for expressing very large numbers.

(280 BCE - 220 BCE) Conon — Associate of Archimedes also associated with the Archimedean spiral

(276 BCE - 197 BCE) Eratosthenes — Prime number sieve, earth measurements.

(262 BCE - 190 BCE) Apollonius — Extensive work on properties of conic sections, use of epicycles.

(190 BCE - 120 BCE) Hipparchus — Early work on trigonometry, use of latter in astronomy.

(190 BCE - 120 BCE) Hypsicles — Wrote Book XIV in *Elements*

(10 - 75) Heron — Area of triangle expressed in terms of sides.

(70 - 130) Menelaus — Spherical geometry.

(85 - 165) Claudius Ptolemy — Trigonometric computations, astronomy.

(200 - 284) Diophantus — Equations over the integers and rational numbers, shorthand (syncopated) notation for expressing algebraic concepts.

(290 - 350) Pappus — Commentaries on earlier work and anthologies of such work, Centroid Theorem(s) for areas and volumes of surfaces and solids of revolution.

(335 - 395) Theon — Influential editing of the *Elements*, commentaties.

(411 - 485) Proclus — Commentaries on earlier work, particularly important because of extensive historical information about Greek mathematics before Euclid.

(476 - 550) Aryabhatta — Base ten numbering system mentioned in his work, introduction of trignometric sine function, more extensive and accurate tables of trigonometric functions.

(480 - 540) Eutocius — Commentaries publicizing the work of Archimedes.

(598 - 670) Brahmagupta — Base ten numbering system explicit, free use of negative and irrational numbers, zero concept included, work on quadratice number theoretic equations over the integers, some shorthand notation employed.

(790 - 850) al-Khwarizmi — Influential work on solving equations, mainly quadratics, beginning of algebra as a subject studied for its own sake.

(836 - 901) Thabit ibn Qurra — Original contribution to theory of amicable number pairs, extensive work translating Greek texts to Arabic.

(940 - 998) Abu'l-Wafa — Highly improved trigonometric computations, discussion of the mathematical theory or repeating geometric designs.

(953 - 1029) Al-Karaji — Introduction of higher positive integer exponents and negative exponents, recursive proofs of formulas that anticipate the modern concept of mathematical induction.

(1048 - 1122) Khayyam — Graphical solutions of cubic equations using intersections of circles and other conics.

(1114 - 1185) Bhaskara — Extremely extensive and deep work on number theoretic questions including solutions to certain quadratic equations over the integers.