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Math 153
Spring 2005
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UPDATED GENERAL INFORMATION — JUNE 1, 2005

Here are things to think about for the final examination, which will take place **11:30 A. M.** on **Thursday, June 9, 2005.** Nominally the examination period last three hours, but the exam will have about twice as much material as either midterm. As noted in class, 55 per cent will be problems and 45 per cent will cover history.

ADDITIONAL OFFICE HOURS. I will be in my office **Monday, June 6,** from **3:00 – 4:30 P. M.** and later if there is demand (not all questions answered or if I know for sure someone is coming later in the afternoon). The usual ways of reaching me are also options.

On the mathematical problems side, here are some suggestions.

(1) Understand the concept of expanding a fraction in the Egyptian form as a sum of unit fractions and how to do so in simple cases.

(2) Know how to work with simple Diophantine equations and find integer solutions, particularly in the quadratic case. In particular, given a Pell's equation like $u^2 = 1 + 3\,v^2$ know how to show that if (u, v) solves the equation then so does $(u + 3\,v^2,\ 3uv)$ and know how to use this and the solution $(7, 4)$ to generate three additional solutions.

(3) Know what it means to say that three perfect squares are consecutive terms in an arithmetic progression and be able to tell whether or not this is true for a given sequence of three perfect squares.

(4) Know how to verify that a specified construction with lines, circles and conic sections solves a given cubic equation (the methods of Manaechmus and Omar Khayyam).

(5) Know how to change variables in a cubic or quartic equation to eliminate the x^2 or x^3 term. Know how to find the integral roots of a polynomial with integral coefficients. See what happens if you apply the change of variables to $x^3 + 3\,x^2 - 9\,x + 5 = 0$.

(6) If $y(x)$ is the Napier logarithm of x, what is the derivative $y'(x)$? What is the indefinite integral?

On the historical side, here are some comments on the types of questions to be asked.

The examination will only cover topics that were not covered in the midterm examination, so it will cover the period beginning with Fibonacci and ending with Cauchy and Weierstrass.

There will be some matching questions similar to the final problem on the second midterm, but the descriptions may involve a 100 year period during which the person made contributions to mathematics. For example, "Chuquet" would match to "15$^{\text{th}}$ century mathematician."

There will also be individual multiple choice questions. Here are two examples:

1. Which of the following mathematical advances is NOT associated with Viète?

A. Increased understanding of trigonometric functions and identities.

B. Modern mathematical symbolism for equality and ddition.

C. Use of letters to denote both known and unknown quantities.

2. Who came first?

A. Cauchy

B. Leibniz

The historical review begins below.

Historical summary

(1170 – 1250) Fibonacci — Introduction of Hindu-Arabic numeration to nonacademics, work on number theory including Fibonacci sequence, problems involving sequences of perfect squares in an arithmetic progression, Pythagorean triples.

(1201 – 1274) al-Tusi, Nasir — Early work on making trigonometry a subject in its own right.

(1219 – 1292) Bacon, Roger — Advocate for putting new mathematical discoveries to practical use.

(1220 – 1280) al-Maghribi — Commentaries on the apocryphal Books XIV and XV of Euclid's *Elements*.

(1225 – 1260) Jordanus — Limited use of letters, results on perfect versus nonperfect numbers.

(1285 – 1349) Ockham — Formulation of the concept of a limit, principle of expressing things as simply as possible (Ockham's razor).

(1313 – 1373) Heytesbury — Mean speed principle for uniformly accelerated motion.

(1323 – 1382) Oresme — Summations of certain infinite series, early ideas on the graphical representation of functions.

(1350 – 1425) Madhava — Infinite series formula for inverse tangent.

(1377 – 1446) Brunelleschi — First specifically mathematical study of drawing in geometric perspective.

(1380 – 1450) al-Kashi — Free use of decimal fractions.

(1401 – 1464) Cusa, Nicholas of — Early mention of cycloid curve, other contributions.

(1404 – 1472) Alberti — first written treatment of geometric perspective theory.

(1412 – 1492) Francesca — Most mathematical treatment of perspective during this time period.

(1412 – 1486) al-Qalasadi — Early versions of some modern notational conventions.

(1436 – 1476) Regiomontanus — Numerous translations of classical works, definitive account of trigonometry as a subject in its own right.

(1445 – 1500) Chuquet — Early versions of some modern notational conventions.

(1462 – 1498) Widman — First appearance of plus and minus signs.

(1465 – 1526) Ferro — Discovery of the cubic formula.

(1471 – 1528) Dürer — Research and writings on geometric perspective.

(1499 – 1545) Rudolff — Introduction of the radical sign $\sqrt{\ }$.

(1500 – 1557) Tartaglia — Independent derivation of cubic formula, extension to other cases.

(1501 – 1576) Cardan — Major work on algebra including cubic and quartic formula, phenomena involving complex numbers.

(1510 – 1558) Recorde — Introduction of an early form of the equality sign.

(1522 – 1565) Ferrari — Quartic formula for roots of a $^{\mathrm{th}}$ degree polynomial.

(1526 – 1573) Bombelli — Use of complex numbers, clarification of cubic formula in the so-called irreducible case.

(1540 – 1603) Viète — Major advances in symbolic notation including the use of letters for known and unknown quantities, results in the theory of equations, new insights into the properties of trigonometric functions and their identities, influential ideas and results about using algebraic methods to study geometric questions.

(1548 – 1620) Stevin — Popularization of decimals throughout Europe, work on centers of gravity, hydrostatics.

(1550 – 1617) Napier — Invention of logarithms.

(1552 – 1632) Bürgi — Independent invention of logarithms, findings published later than Napier and Briggs.

(1560 – 1621) Harriot — Introduction of symbolism in his works (modern inequality signs first appear her, inserted by editors).

(1561 – 1615) Roomen — Formulation of challenging algebraic problem solved by Viète.

(1561 – 1630) Briggs — Continued Napier's work and published tables of common base 10 logorithms.

(1564 – 1642) Galileo — Important examples of curves arising from moving objects, Galilean paradox regarding infinite sets.

(1571 – 1630) Kepler — Laws of planetary motion, use of infinitesimals to find areas, Wine Barrel Problem in maxima and minima.

(1574 – 1660) Oughtred — Invention of $\times$ for multiplication, invention of the slide rule.

(1577 – 1643) Guldin — Rediscovery of Pappus' Centroid Theorem.

(1584 – 1667) Saint-Vincent — Integral of $1/x$, refutation of Zeno's paradoxes using the concept of a convergent infinite series.

(1595 – 1632) Girard, Albert — Trigonometric notation, formula for area of a spherical triangle.

(1596 – 1650) Descartes — Refinements of Viète's symbolic notation including the use of x, y, z for unknowns, introduction of coordinate geometry in highly influential publication *Discours de la méthode*, but not including key features like rectangular coordinates or many of the standard formulas. The work on coordinate geometry was greatly influenced by classical Greek geometers such as Apollonius and Pappus and also by the work of Viète.

(1598 – 1647) Cavalieri — Investigations of areas and volumes, Cavalieri's cross section principle(s), integration of positive integer powers x^n by geometric means.

(1601 – 1665) Fermat — Important insights in number theory, coinventor of coordinate geometry (closer to the modern form than Descartes in many respects), preliminary work aimed at describing tangent lines and solving maximum and minimum problems. The work on coordinate geometry was greatly influenced by Apollonius in some respects and Viète in others.

(1602 – 1675) Roberval — Motion-based definition of tangents, numerous results on cycloids.

(1608 – 1647) Torricelli — Computations of integrals, results on cycloids, discovery of solid of revolution that is unbounded but has finite volume.

(1616 – 1703) Wallis — Free use of nonintegral exponents, extensive integral computations including x^r where r is not necessarily a positive integer, major shift to algebraic techniques for evaluating such integrals.

(1620 – 1687) Mercator, N — Standard infinite series for $\ln(1 + x)$.

(1622 – 1676) Rahn — First use of the standard division symbol $\div$.

(1623 – 1662) Pascal, Blaise — Many important contributions, including properties of cycloids and integration of $\sin x$.

(1628 – 1704) Hudde — Free use of letters to denote negative numbers, standard formulas for slopes of tangent lines to polynomial curves.

(1629 – 1695) Huygens — Numerous contributions, including solution of Galileo's isochone problem.

(1630 – 1677) Barrow — More refined definition of tangent line, realization that differentiation and integration are inverse processes, integrals of some basic trigonometric functions.

(1633 – 1660) Heuraet — Mathematical description of arc length and computations for some important examples.

(1638 – 1675) Gregory, James — Integration of certain trigonometric functions, familiar power series for the inverse tangent, first attempt to write a textbook on advances leading to calculus.

(1640 – 1718) La Hire — Work on solic analytic geometry and other aspects of geometry.

(1643 – 1727) Newton — Coinventor of calculus. Formulation of earlier work in more general terms, recognition of wide range of applications, material exressed in algebraic as opposed to geometric terms. Main period of discovery in 1660s, publication much later. Work strongly linked to his study of physical problems, particularly planetary motion. His main work on the latter, *Principia*, was highly mathematical. He obtained the standard binomial series expansion for $(1+x)^r$, where r is real. Notation for calculus included *fluxion* for derivative, *fluent* for integral and $\dot{x}$ for the derivative. Infinitesimals were not strongly emphasized, but the use of infinite series to express functions was stressed. Priority was placed on differentiation. Newton's applications of calculus was extremely important influence in determining the subsequent development of mathematics for well over a century.

(1646 – 1716) Leibniz — Coinventor of calculus. Formulation of earlier work in more general terms, recognition of wide range of applications, material exressed in algebraic as opposed to geometric Main period of discovery in 1670s, published in the next decade. Infinitesimals were strongly emphasized. The Leibniz notation, including dy/dx for derivative and $\int\ y\,dx$ for integral, became standard. Emphasis wason finding solutions that could be written in finite terms rather than infinite series. Priority was placed on integration. Leibniz also made extremely important contributions to philosophy.

(1654 – 1705) Bernoulli, Jacob — Continued work on calculus and differential equations as well as many other important contributions.

(1661 – 1704) de L'Hospital — Publication of calculus book with formula bearing his name (purchased from Johann Bernoulli).

(1667 – 1748) Bernoulli, Johann — Continued work on calculus and differential equations as well as many other important contributions.

(1667 – 1748) de Moivre — Polar form of complex numbers $re^{i\theta} = \cos\theta + i\,\sin\theta$, also other important work.

(1685 – 1753) Berkeley — Extremely influential critique of infinitesimals in calculus ("ghosts of departed quantities").

(1685 – 1731) Taylor, Brook — Publication of series expansion and approximation formulas bearing his name.

(1698 – 1746) Maclaurin — Publication of previously known power series expansion bearing his name, geometrical studies, lengthy response to Berkeley phrased in classical geometric terms.

(1707 – 1783) Euler — Extremely important contributions to many areas of mathematics, including number theory, infinite series and solid analytic geometry.

(1713 – 1765) Clairaut — Development of solid analytic geometry, other contributions.

(1717 – 1783) d'Alembert — First suggestion of a concept of limit to circumvent logical problems with infinitesimals.

(1765 – 1802) Ruffini — First effort to prove that no quintic (5^{th} degree) formula exists.

(1777 – 1855) Gauss — Extremely important contributions to many areas of mathematics.

(1789 – 1857) Cauchy — Matheamtical definition of limit in 1820 – nearly 150 years after the publication of Leibniz' work, also many other important contributions.

(1802 – 1831) Abel — Improved argument that radical formulas for roots of polynomials with degree ≥ 5 do not exist, insistence on a logically rigorous development of infinite series, other extremely important and far-reaching contributions over a very short lifetime.

(1815 – 1897) Weierstrass — The modern $\varepsilon - \delta$ definition of a limit, also many other important contributions.

(1831 – 1916) Dedekind — Mathematically rigorous description of the real number system, also many other important contributions.

(1845 – 1918) Cantor, Georg — Theory of infinite sets (the logical foundation of modern mathematics).

(1918 – 1974) Robinson, Abraham — Logically rigorous formulation of infinitesimals (non-standard analysis)