0.A. Additional background discussion

In this supplement to the Introduction we shall discuss a few assorted issues of varying importance. As in other areas of knowledge, there are many commonly repeated stories about mathematical history which are often presented as historical facts but are speculative, biased or completely false. Sometimes such misinformation is harmless and can even stimulate a great deal of interest in mathematics and its history, but in other cases it can lead to incorrect conclusions that are inaccurate, unfair or even harmful. We shall pay particular attention to the reliability of some printed sources and references that are cited throughout these course notes, and we shall also discuss the quality of various Internet references, most notably ***Wikipedia*** articles and the results of ***Google*** searches. In the Introduction, we have already mentioned some other issues which must be taken into account, like the setting of boundaries for the main subject matter of this course and gaps in our current knowledge, especially when one looks beyond the past thousand years or so. Specific examples of the latter will be mentioned at many points throughout the notes, so we shall focus on general points to keep in mind throughout this course. Finally, we shall discuss the level(s) at which we discuss specific mathematical issues and some of the stylistic conventions which have been adopted here.

Comments about selected references

There are many different types of history books. They are written for a variety of reasons and goals, some of which are clearly good, some of which are not so good, others of which are clearly stated, and still others of which are suppressed, either deliberately or not consciously. Some books are mainly chronologies of events and noteworthy individuals, and others are analyses or reconstructions of life at some earlier times and places. Usually history books are combinations of all such features, with an attempt to strike a balance between presenting specific facts and providing general analyses. Not surprisingly, achieving such a balance is often challenging, and most books are stronger in some respects than in others. The challenges are even greater when one attempts to cover dozens of centuries and many different cultures in a single volume or series of volumes, as is the case with most texts on the history of mathematics. Therefore we shall include some comments about a few references for the history of mathematics that are cited in this course. These comments are intended to reflect current "mainstream" mathematical and historical thought, at least to the extent that one individual can state them "objectively."

We are particularly interested in two issues:

<u>Historical accuracy.</u> All of us have heard interesting, frequently repeated stories about historical figures which are sometimes presented as historical facts even though they might not be. Often these reflect some historical reality, and if one recognizes their fictional or legendary nature they can lead to a better understanding or appreciation of history. The following quotation illustrates this point:

> History books that contain no lies [or embellishments] are extremely dull.
>
> Anatole France (Jacques Anatole François Thibault, 1844 – 1924),
> *The Crime of Sylvestre Bonnard* (1881).

On the other hand, as noted above, fictional or legendary stories can also lead to seriously inaccurate, or even dangerous, conclusions, and ultimately the subject must be based upon factual information. There are many examples of mathematical anecdotes which are often stated as facts but are either highly questionable or provably false, and some of the better known examples are discussed in Burton or in these notes.

<u>Controversial agendas.</u> The news media constantly provide examples of individuals or groups who effectively write their own versions of history with the intention of justifying their views. In some cases there are deliberate or malicious falsifications, but in others such misrepresentations may simply reflect sincere but highly questionable views. Frequently historical facts and strong, highly debatable opinions appear right next to each other in such writings, and it is not always easy to determine just where facts end and propaganda begins. If a reader is aware of a writer's controversial views, it is often helpful regardless of whether the reader agrees or disagrees with such views.

We shall consider some examples here. In each case we shall consider books that have some extremely good features although other features might be problematic.

Since we have already mentioned two of Morris Kline's numerous works on the history of mathematics and his books have very high profiles in the subject, we shall start with some comments on his writings. Kline's books contain a great deal of information and many extremely well – written passages. However, his very strongly negative opinions on 20th century mathematics (by implication, 21st century mathematics as well**!**) were extremely controversial, and although they are often presented quite attractively and forcefully, a closer examination of the facts often indicates serious difficulties with many crucial points. Fortunately, such controversial opinions are rarely stated explicitly or disruptively in his historical writings, and this applies particularly to the portions about the development of the subject through the end of the 19th century. One should be aware of these when reading his strongly negative comments about 20th century mathematics or the mathematical legacies of certain ancient or non – Western cultures. More will be said about this towards the end of the course.

At this point it also seems appropriate to mention a widely read classic, which for three quarters of a century has been a very influential popularization of mathematical history. Several prominent mathematicians have cited it as a key motivation for their decisions to work in the subject.

> **E. T. Bell*, Men of Mathematics.*** *Touchstone Books (Simon and Schuster), New York,* 1986. **ISBN:** 0–671–62818–6.

This book is beautifully written and it is an extremely fascinating piece of literature, but its historical scholarship is extremely (perhaps dangerously) inaccurate in many places, including some of the best known chapters. In particular, its treatments of some controversies involving mathematicians are definitely one – sided. To summarize, this book is definitely worth reading, but a reader should be aware of its deficiencies.

Given the size of many textbooks on the history of mathematics and the extent of their coverage, it is not surprising that thorough reviews of them (say, a few pages long by qualified, objective reviewers) generally do not exist. One summary of published reviews is the following online site which was quoted in the Introduction:

<u>http://www.math.usma.edu/people/Rickey/hm/mini/textbooks.html</u>

Of course, the web pages for such books on the **http://www.amazon.com** site also contain reviews of the most widely used books, but as usual for such Internet forums the quality of such reviews is highly variable, and for most books on the history of mathematics the number of reviews is too small to yield more than fragmentary information.

Very long and comprehensive texts, like Boyer and Merzbach or Katz, cover enormous amounts of material and are generally fairly accurate, but given their lengths and breadths it is not surprising that the contain some noteworthy factual errors; a few are listed in the Internet reviews, and there are also some others (for example, the misuse of the term *abacists* in Katz). The books by Calinger, Eves and Burton may be better in this respect, but this may simply reflect an absence of detailed reviews. In all cases the books are pretty reliable overall accounts of the history of mathematics, but it is usually worthwhile to check independent sources to confirm the details.

The short book by D. Struik (1894 – 2000) is an extremely singular case. His Marxist political views (in the classical rather than Soviet sense) were well – known, but his book certainly does not politicize the subject (in contrast, some books on the subject from Eastern Europe during the Cold War from 1945 to 1990 contain repeated, gratuitous references to K. Marx and F. Engels). Struik's book does a really remarkable job of surveying the history of mathematics very accurately and effectively in a relatively small number of pages, but of course some important developments — and many significant details — are missing. There are two noteworthy features of the book that were highly innovative when the First Edition appeared in 1948; namely, there were concerted efforts to say more about the mathematical legacies of non – Western cultures (with still further efforts in subsequent editions) and to relate developments in mathematics to the everyday lives of people at the time (going beyond the political, economic or intellectual elites). An interesting review of this classic work is available online at the following address:

http://www.ams.org/notices/200106/rev-rowe.pdf

Internet resources

Traditional printed publications in mathematics are normally filtered through an editorial reviewing process which checks their accuracy (not perfectly, but for the most part very reliably). Some widely used Internet sources maintain similar standards (for example, most of the sites supported by recognized academic institutions), but others have far more lenient standards, and this fact must be acknowledged. Probably the most important single example is the widely used ***Wikipedia*** site:

http://en.wikipedia.org/wiki/Main_Page

The ***Wikipedia*** site contains an incredibly large number of articles, with extensive information on a breathtakingly vast array of subjects. The articles are written by volunteers, and in most cases they can be edited by anyone with access to the Internet, including some individuals whose views or understanding of a subject may be highly controversial or simply unreliable. This issue has been noted explicitly by ***Wikipedia*** in its articles on itself, and in particular the following discuss the matter in some detail.

http://en.wikipedia.org/wiki/Wikipedia

http://en.wikipedia.org/wiki/Reliability_of_Wikipedia

Since these notes make numerous references to ***Wikipedia*** articles, the underlying policies and reasons for doing so deserve to be discussed. First of all, despite the justifiable controversy surrounding the reliability of some online ***Wikipedia*** articles, the entries for standard, well – established topics in the sciences are generally very reliable, and the ones cited in the course notes were specifically checked for accuracy before they were cited. As such, in these notes they are inserted as convenient but reliable online alternatives to more traditional library references strictly on a case by case basis. In most if not all cases where the accuracy of ***Wikipedia*** articles can be questioned (generally in articles about history), we have noted this point. In particular, our citations in these notes should not be interpreted as a blanket policy of acceptance for all such articles, even in the sciences. Generally speaking, it is best to think of ***Wikipedia*** articles as merely first steps in gathering information about a subject and not as substitutes or replacements for more authoritative (printed or electronic) references in term papers or scholarly articles. **ALL** statements in ***Wikipedia*** articles definitely should be checked independently using more authoritative sources, especially when writing papers for class assignments or formal publication.

In any discussion of Internet references, some comments about World Wide Web searches using ***Google*** (or other search engines) are also appropriate. The extreme popularity and wide use of ***Google*** searches clearly show their value for all sorts of purposes. Of course, it is important to remember that search engines are designed to make money and that profit motives might affect the results of searches and the order in which sources are listed, but usually this is not a problem for topics in history or the sciences. Most of the time search engines are extremely reliable at listing the best references first, but this is not always the case, and therefore it is recommended that a user should normally go beyond the first page of **10** search results. As a rule, it is preferable to look at the top **20, 50** or even **100** results.

Influences of different cultures upon each other

There are some obvious instances in which the mathematics developed in one culture has had a major impact upon the mathematics of another, and these are discussed in the text and the notes. However, there are also numerous speculations about even further interactions between cultures and their impacts. For example, there are questions about the extent to which ideas from Greek mathematics migrated to China, and similarly about the extent to which ideas from Chinese mathematics made their way to India and vice versa. These are intriguing questions along these lines, and it is very conceivable that some exchanges took place, but usually there is not enough evidence to draw firm conclusions, and in any case the different cultures clearly made important advances in separate directions.

The level(s) of mathematical discussions

The prerequisite for this course is first year calculus, so ideas from high school and college courses through precalculus and calculus will be used freely. At a few points we shall also use input from other lower division undergraduate courses, most notably the basics of multivariable calculus (Mathematics **10A – B**) or discrete mathematics (Mathematics **11**). In the latter case, many of the topics are already covered at least lightly in high school course, and we shall try to minimize the background material that we use, but concepts from discrete mathematics are unavoidable in discussions related to number theory and the development of algebra, so such points cannot be avoided entirely. Frequently we have included discussion of more advanced topics simply for the sake of completeness due to a lack of readily accessible references.

There will also be some places in the notes where the mathematical discussions are at higher levels (usually at the advanced undergraduate level, but in a few cases more is needed). However, usually these are only included for the sake of mathematical completeness, particularly in situations where it is difficult or impossible to provide good references to other sources, and in such instances it will not be necessary to understand the details. Similar considerations apply to many of the references in the notes for mathematical topics; if the material seems too difficult, then one should focus on the main statements and not worry about understanding the proofs or derivations.

Various stylistic conventions

In many cases, there are several different renderings of a mathematician's name in the literature, even if one only considers material written in English. Generally we have chosen forms of names which are sufficiently widely used in the English language that they are easily recognized, and in borderline cases we have chosen recognizable forms which are as close as possible to the names in the original languages. Of course, the further one goes from European names the more difficult this is, and it is especially difficult with Chinese names from earlier times because there are several frequently used but extremely different transliteration systems; in the latter case we generally give both the standard "old" and "new" transliterations whenever possible.

Finally, we should explain the reasons behind our conventions of B. C. E. and A. D. for historical dating. The standard *Anno Domini* numbering convention for historical dates apparently goes back to about 525 (A. D., of course), but no reasons were given for deciding upon this number when it originally appeared the writings of Dionysius Exiguus (*c.* 470 – *c.* 544). Most of the independent historical evidence suggests that the actual birth of Jesus Christ took place at some time before the year 1 A. D., but other evidence indicates a later date, and to avoid these issues it seems more accurate to use B. C. E. (*Before the Common Era*) rather than B. C. (*Before Christ*). On the other hand, we have chosen to write A. D. rather than C. E. (*Common Era*, or *Christian Era*) because the current use of such dates (or even the initials themselves) does not necessarily reflect an individual's religious preferences any more than the use of standard names for months or days of the week which came from the ancient Roman or Germanic religions, and it also seemed that the use of contrasting initials might eliminate potential sources of errors or confusion (there have been somewhat whimsical remarks suggesting the initials should stand for *Arbitrary Demarcation*).

For the sake of completeness, we note that the year 1 A. D. comes immediately after the year 1 B. C. E.; this might not be surprising since there was no concept of zero in classical Greek mathematics, but there is some conflict between the usual historical conventions and the standard system for numbering of dates in astronomy, in which the standard calendar year n B. C. E. becomes the astronomical year $1 - n$.