Other contributions of Newton and Leibniz

Although there are clear differences in the Newton and Leibniz approaches to calculus, each perspective has advantages in certain situations. For example, some functions cannot be expressed in finite terms involving standard functions from first year calculus but can be studied very effectively using infinite series as in Newton's treatment, but others cannot be studied using infinite series and thus are more compatible with Leibniz' viewpoint. Examples of each type appear in **http://math.ucr.edu/~res/history14c.pdf** and **http://math.ucr.edu/~res/history14d.pdf**.

Both Newton and Leibniz made other substantial contributions to mathematics aside from their momentous work on calculus. We shall mention a few of these contributions here.

Newton — Codiscovered (with J. Raphson, 1648 – 1715) the ***Newton – Raphson method*** for numerically approximating solutions to complicated nonlinear equations. This is one of the best known methods for finding such solutions; there are descriptions of this method in many standard calculus textbooks. Examples of the method and a brief description are given on page 434 of Burton.

Newton — In an appendix to his major scientific work titled ***Opticks*** (published in 1704), he classified all curves in the coordinate plane which are defined by third degree polynomial equations in the coordinates x and y. This has been characterized as the first comprehensive study of curves and their properties since the work of Apollonius. Newton also did several other important pieces of algebraic work, including his generalization of the binomial formula (which we have already mentioned) and his codiscovery (with A. Girard) of certain important identities involving symmetric polynomials in n algebraically independent variables (a polynomial $p(x_1, \ldots, x_n)$ is said to be ***symmetric*** if any permutation of the variables yields the same polynomial; for example, $x_1 + \ldots + x_n$ and $x_1 \ldots x_n$ are symmetric but x_1 and $x_1x_3 + x_2x_3$ are not). Here are some references for further information:

http://en.wikipedia.org/wiki/Cubic_plane_curve

http://mathworld.wolfram.com/CubicCurve.html

http://www.2dcurves.com/cubic/cubic.html

http://en.wikipedia.org/wiki/Newton%27s_identities

http://staff.jccc.net/swilson/planecurves/cubics.htm

Newton — Contributions to the calculus of finite differences, whose uses include describing recursively defined sequences like the Fibonacci numbers. Further information on finite differences and Newton's results can be found at the following online site:

http://en.wikipedia.org/wiki/Finite_differences

Leibniz — Early work on studying logic using algebraic methods to manipulate well – formed grammatical statements. This has been described as one of the most important advances in the theory of logic between the time of Aristotle and the more systematic work on symbolic logic in the 19th century by G. Boole (1815 – 1864) and others. Further information on the development of algebraic methods in logic can be found on pages 7 – 8 of the following:

http://math.ucr.edu/~res/math144/setsnotes1.pdf

On page 8 of this document there is a reference to pages 643 – 647 in the 6th Edition of Burton; the corresponding pages in the 7th Edition are 646 – 650.

Leibniz — A comprehensive description of the ***binary*** or base 2 numeration system. This expanded upon many earlier ideas about representing numbers or words by combinations of two symbols, including numeration systems in a few cultures and the ancient writings of Pingala, and in fact Leibniz explicitly cited symbolism in the ancient Chinese ***I Ching*** (in modern Pinyin transliteration, **Yì Jīng**) as one forerunner of his work. The importance of binary numeration for electronic computing was recognized beginning in the nineteen thirties, most notably by C. Shannon (1916 – 2001). Here are some online references for further information:

http://en.wikipedia.org/wiki/I_Ching

http://en.wikipedia.org/wiki/Binary_numeral_system

http://www.math.grin.edu/~rebelsky/Courses/152/97F/Readings/student-binary

http://www.mathsisfun.com/binary-number-system.html

http://www.thocp.net/biographies/shannon_claude.htm

http://en.wikipedia.org/wiki/Claude_Shannon

Leibniz — Solutions of systems of linear equations, including the definition of the determinant of an square matrix and the standard formula for it as a polynomial function of its entries (K. Seki did similar things independently about one decade earlier).