Math 153
Spring 2010
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EXERCISES RELATED TO `history05.pdf`

As in the earlier exercises, "Burton" refers to the Seventh Edition of the course text by Burton (the page numbers for the Sixth Edition may be off slightly).

- Burton, p. 231: 2, 4, 6, 8, 10, 12, 13, 14, 15
- Burton, p. 237: 1–2

Additional exercises

Solve the first three using Pappus' Centroid Theorem(s):

1. What are the centers of mass for a semicircular wire $x^2 + y^2 = 1$, $y \geq 0$ and the solid half disk defined by $x^2 + y^2 + \leq 1$, $y \geq 0$? Which is closer to the origin? [*Hint:* What surface and solid of revolution does one obtain by rotating these objects around the x-axis?]

2. The area of an ellipse with major axis of length a and minor axis of length b is $\pi\, ab$, and the volume of an ellipsoid with principal axes of lengths a, b and c is $\frac{4}{3}\pi\, abc$. Find the center of mass for the half ellipse defined by the inequalities $b^2x^2 + a^2y^2 \leq a^2b^2$ and $y \geq 0$.

3. Find the volume of the solid of revolution formed by rotating the $3, 4, 5$ right triangle with vertices $(0, c)$, $(3, c)$ and $(0, c + 4)$ about the x-axis.

4. The equation $x^2 = 7y + r$ for $r = 1, 2, 3, 4, 5, 6$ can be solved for three values of r and cannot be solved for the other three values. Find the values for which it can be solved and for which it cannot be solved. In the cases where it can be solved, all solutions have the form $7z \pm s$ for some value of s between 1 and 6. In each case where the equation $x^2 = 7y + r$ has an integral solution, find the associated value of s.

5. Suppose that we are given a right triangle $\triangle ABC$ in the coordinate plane whose vertices are $A = (0, a)$, $B = (b, 0)$ and $C = (0, 0)$. Let F and G denote the points $(-a, 0)$ and $(-a, a)$, so that A, C, F, G form the vertices of a square (in the given order), and H and I denote the points $(0, -b)$ and $(-b, -b)$, so that B, C, H, I also form the vertices of a square (in the given order). Prove that the point P where FG and HI meet has coordinates $(-a, -b)$, and also prove that the line PC is perpendicular to the hypotenuse AB. [*Hint:* Show that one equation for AB is given by $ax + by = ab$, and one equation for the line PC is given by $bx - ay = 0$. Why does this imply perpendicularity?]

REMARK. The conclusion for this exercise figures in the discussion of the Pappus Area Theorem in `history05f.pdf`, which also contains one further exercise motivated by that result.