

Quiz 3 Suggestions

The quiz will concern solutions to Diophantine equations of the form $a^p + b^q = c^r$ where $a, b, c > 0$ are rational or integral and $p, q, r \geq 2$ are positive integers. Here are ~~two~~ ^{three} practice problems.

1. Let b be a positive rational number. Show that there are infinitely many (a, c) such that $a^2 + b^3 = c^2$. [Hint: $b^3 = c^2 - a^2$. Let u, v ($u > v$) satisfy $wv = b^3$, and find (a, c) such that $a + c = w$, $c - a = v$.]
2. Let k be any odd positive integer > 1 . Show that there is a solution to the equation $a^k + b^k = c^2$ where a and b and c are positive integers. [Hint: $2^3 + 2^3 = 4^2$.]
3. Suppose that $a^2 + b^2 = c^2$ where a, b, c are positive integers. Find a positive integer k such that $(ka)^2 + (kb)^2 = d^3$ for some positive integer d . [Hint: Express the left side in terms of k and c .] Try $k = c^n$ for some n .