Quiz 3 MATH 153, SPRING 2010

SECTION (OR TA): NAME:

Let b be a positive integer greater than 1, and write $b^3 = uv$ where u and v are positive integers such that $u > v$ and u, v are both even or both odd.

(i) If $c > a > 0$ are integers such that $a + c = u$ and $c - a = v$, explain why $a^2 + b^3 = c^2$.

Under these conditions we have

$b^3 = vu = (c-a)(c+a) = c^2 - a^2$, so that

$a^2 + b^3 = c^2$.

(ii) Find two positive integral solutions of $a^2 + b^3 = c^2$ with $b = 6$.
Hint: Factor $6^3 = 216$ *two ways.*

Follow the hint. $6^3 = 2^3 \cdot 3^3 = 6^2 \cdot 6 = 36 \cdot 6 =$

$(2^2 \cdot 3) \cdot (2 \cdot 3^2) = 12 \cdot 18$

First solution:

$36 = c + a$

$6 = c - a$

Second solution:

$18 = c + a$

$12 = c - a$

SOLVE FOR c and a.

$c = 21$, $a = 15$

$c = 15$, $a = 3$

HENCE

$15^2 + 6^3 = 21^2$

$3^2 + 6^3 = 15^2$

CHECK

$225 + 216 = 441$

$9 + 216 = 225$

Quiz 3

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SECTION (OR TA): NAME:

Suppose that a, b, and c are positive integers such that $a^3 + b^3 = c^2$.

(i) If k is a positive integer, show that there is some $d > 0$ such that $(k^2a)^3 + (k^2b)^3 = d^2$.

Write out the left hand side:

$(k^2a)^3 + (k^2b)^3 = k^6a^3 + k^6b^3 = k^6(a^3+b^3) =$

$k^6c^2 = (k^3c)^2$ so we can take $d = k^3c$.

!! TYPO

(ii) Using (i) and $1^3 + 2^3 = 3^2$, $2^3 + 2^3 = 4^2$, find two solutions to $a^3 + b^3 = c^2$ of the form (a', b_0, c'), (a'', b_0, c'') for some $b_0 > 2$.

Put $k = 2$ in the preceding discussion, so that $b_0 = 8$ and we obtain the following:

$4^3 + 8^3 = 24^2$ $(64 + 512 = 576)$

$8^3 + 8^3 = 32^2$ $(512 + 512 = 1024)$

Actually, in the first case one obtains a solution to $x^2 + y^3 = z^2$: Take $x = y = 8$, $z = 24$.

REMINDER FROM THE LECTURES

Integral solutions to $a^2+b^4=c^2$, where $a, b, c > 0$.

Easy example: $5^2 = 3^2 + 4^2 = 3^2 + 2^4$.

Systematic families: Start with a Pythagorean triple $(a, b; c)$. Then

$$(bc)^2 = b^2c^2 = b^2(a^2+b^2) = b^2a^2+b^4 = (ab)^2+b^4.$$

SOLUTIONS TO $a^2+b^k=c^2$, $k \geq 3$ arbitrary ($a, b, c > 0$ again).

Here is a simple way to find ∞ families for each k: Start with

$$n^2 + (2n+1) = (n+1)^2$$

and observe that, for each $k \geq 3$, there are infinitely many choices of n such that $2n+1 = m^k$ for some m.