Math 153
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Preparation suggestions for the first examination

The first midterm will be about 65 per cent problems and 35 per cent historical or short answer. For the historical part the main thing is to know the sequence of important developments and of important figures listed in the notes, basically in order and accurate to about a third of a century. In particular, knowledge of the material in each historic figure's work will be tested; these will include items like Euclid's construction of perfect numbers in the *Elements*, his inclusion of proportionality theorems which also cover the irrational case, Archimedes' study of the spiral curve, Hippocrates' result on the areas bounded by lunes, and the Pythagorean's discovery of the irrationality of $\sqrt{2}$. There may also be questions for which the answer is "none of the above," sometimes because the given statement is false. Here are a few examples:

Construction of a regular polyhedron with hexagonal faces [no such figure exists].

Proof that the vertices of a regular tetrahedron lie in a single plane [they don't].

Constructed two odd perfect or infinitely many perfect numbers [in the first case none are known to exist; in the second, it is not known if there are infinitely many perfect numbers].

For the mathematical part, the problems in the exercises are good practice material. Also, a thorough understanding of background material like elementary algebra and geometry, precalculus, and differential calculus will be assumed, and problems drawing upon such knowledge are likely to be on the examination. Some aspects have been mentioned explicitly in the course notes, but a few others may appear. Here are a few additional problems worth working as preparation:

1. In `history02c.pdf` we note that the basic problem with the incorrect attempt at angle trisection is its assumption that $\tan 3x = 3\tan x$. Show that $\tan 3x - 3\tan x$ is positive when $x > 0$ and x is sufficiently small; since the value of this function at $x = 0$ is 0, it suffices to show that the function's derivative is positive if $0 < x <$ (say) $\frac{1}{6}\pi$. Why does this suffice, and why is the assertion true?

2. Let $c > 0$ be a real number. For which values of c do the circles $x^2 + y^2 = 1$ and $(x-c)^2 + y^2 = 4$ meet in 0, 1 and 2 points? In each case, find all the intersection points.

3. In coordinate space, no line in the plane $z = 1$ meets the x-axis, but through a given point in that plane only one line is parallel to the x-axis. If M is the unique line in this plane through $(0, 0, 1)$ which is parallel to the x-axis, give the coordinates for a second point on M.

4. Suppose that $\triangle ABC$ has a 72 degree angle at A. There are at exactly two choices of x such that $|\angle ABC| = x^\circ$ and $\triangle ABC$ isosceles. Find these values of x.

5. Suppose that we are given the points A, B, C on the circle $x^2 + y^2 = 1$ with coordinates $(0, 1)$, $(-\frac{3}{5}, -\frac{4}{5})$ and $(1, 0)$ respectively. Find $|\angle ABC|$ and explain how one can do this without using the vector formula for angle cosines (look in `history03.pdf`).

6. How many regular polyhedra have faces that are not equilateral triangles? What sorts of faces do they have?

Some of these are related to problems which will appear on the exam (and others came from possible problems that were rejected).