Math 153
Spring 2005
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**Preparation suggestions for the second examination**

The first midterm will be about 65 per cent problems and 35 per cent historical or short answer. For the historical part the main thing is to know the sequence of important developments and of important figures listed in the notes, basically in order and accurate to about a century or slightly less. A review summary is included at the end of this document.

For the mathematical part, the problems in the exercises are good practice material. Also, a thorough understanding of background material like elementary algebra and geometry, precalculus, and first year calculus will be assumed, and problems drawing upon such knowledge are likely to be on the examination. Some aspects have been mentioned explicitly in the course notes, but a few others may appear. Here are a few additional problems worth working as preparation:

**1.** It is possible that some aspect of Archimedes' computation for the volume of a parabolic sector will appear, but perhaps not explicitly.

**2.** There may be something about positive integer or positive rational solutions for polynomial equations.

**3.** Something related to Pappus' Centroid Theorem may appear.

**4.** Something related to Pell's Equation may appear.

**5.** There may be a problem on the Chinese Remainder Theorem or some aspect of it.

**6.** There may be something related to the historical development of trigonometry.

Some of these are related to problems which will appear on the exam (and others came from possible problems that were rejected).

*Material covered on the examination*

Here is the list of lecture notes files in the directory `http://math.ucr.edu/~res/math153` to be studied for the midterm examination:

chineseremainder.pdf
confracs.pdf
diophantus-examples.pdf
history00.pdf
history03.pdf
history03a.pdf
history03b.pdf
history03c.pdf
history03d.pdf
history04X.pdf
history04Y.pdf
history04Z.pdf
history04a.pdf
history04b.pdf
history04c.pdf
history05.pdf
history05a.pdf
history05b.pdf
history05c.pdf
history05d.pdf
history05e.pdf
history06X.pdf
history06Y.pdf
history06b.pdf
history06c.pdf
history06d.pdf
history07.pdf
history07a.pdf
history08.pdf
math153exercises03.pdf
math153exercises04.pdf
math153exercises05.pdf
math153exercises05a.pdf
math153exercises06.pdf
math153exercises06a.pdf
math153solutions03.pdf
math153solutions04.pdf
math153solutions04a.pdf
math153solutions05.pdf
math153solutions05a.pdf
math153solutions06.pdf
math153solutions06a.pdf
oldmagicsquare.pdf

`oldmagicsquare2.pdf`
`pappusproblems.pdf`
`quadcirc.pdf`
`spiralregions.pdf`

The corresponding sections of the text are indicated at the top of the files `history0`$*$`.pdf` listed above (where $*$ is just a number in the range 3–8 with no extra letter).

**GENERAL POLICIES FOR EXAMINATIONS.** Unless indicated otherwise, on the mathematical problem-solving portion exams the logical steps in the written answers should be shown to ensure the maximum possible credit; partial credit will be given for incorrect answers in some cases, depending upon the extent to which the work shown on the exam is valid or indicates insight into the problem.

No electronic computing or communications devices will be necessary, and none will be permitted. As a courtesy to others, all ringers for cell phones should be turned off. Likewise, no open books or notes will be permitted. Limited amounts of additional paper will be available if students are not able to complete the examination on the pages that will be handed out. Information about grading of the examination will be posted when the graded papers are distributed.

**Historical summary**

(428 BCE - 348 BCE) Plato — Influential ideas about how mathematics should be studied.

(408 BCE - 335 BCE) Eudoxus — Proportion theory for irrationals, method of exhaustion to derive formulas. cube using intersecting parabolas.

(380 BCE - 320 BCE) Menaechmus — Early work on conics, duplication of cube using intersecting parabolas.

(325 BCE - 265 BCE) Euclid — Organized fundamental mathmatial material in the *Elements*, including material on geometry, number theory and irrational quantities.

(287 BCE - 212 BCE) Archimedes — Computations of areas and volumes, study of spiral curve, methods for expressing very large numbers.

(310 BCE - 230 BCE) Aristarchus — Heliocentric universe, astronomical measurements, simple continued fractions.

(280 BCE - 220 BCE) Conon — Associate of Archimedes also associated with the Archimedean spiral

(276 BCE - 197 BCE) Eratosthenes — Prime number sieve, earth measurements.

(262 BCE - 190 BCE) Apollonius — Extensive work on properties of conic sections, use of epicycles.

(190 BCE - 120 BCE) Hipparchus — Early work on trigonometry, use of latter in astronomy.

(190 BCE - 120 BCE) Hypsicles — Wrote Book XIV in *Elements*.

(10 AD - 75) Heron — Area of triangle expressed in terms of sides.

(70 - 130) Menelaus — Spherical geometry.

(85 - 165) Claudius Ptolemy — Trigonometric computations, astronomy.

(200 - 284) Diophantus — Equations over the integers and rational numbers, shorthand (syncopated) notation for expressing algebraic concepts.

(290 - 350) Pappus — Commentaries on earlier work and anthologies of such work, Centroid Theorem(s) for areas and volumes of surfaces and solids of revolution.

(335 - 395) Theon — Influential editing of the *Elements*, commentaries.

(410 - 485) Proclus — Commentaries on earlier Greek mathematics and its history.

(476 - 550) Aryabhatta — Base ten numbering system mentioned in his work, introduction of trigonometric sine function, more extensive and accurate tables of trigonometric functions.

(480 - 540) Eutocius — Commentaries publicizing the work of Archimedes.

(598 - 670) Brahmagupta — Base ten numbering system explicit, free use of negative and irrational numbers, zero concept included, work on quadratic number theoretic equations over the integers, some shorthand notation employed.

(790 - 850) al-Khwarizmi — Influential work on solving equations, mainly quadratics, beginning of algebra as a subject studied for its own sake.

(836 - 901) Thabit ibn Qurra — Original contribution to theory of amicable number pairs, extensive work translating Greek texts to Arabic.

(850 - 930) Abu Kamil — Further development of algebra.

(850 - 930) Al-Battani — Work in computational trigonometry and trigonometric identities.

(940 - 998) Abu'l-Wafa — Highly improved trigonometric computations, discussion of the mathematical theory or repeating geometric designs.

(950 - 1009) Ibn Yunus — Trigonometric computations and identities.

(953 - 1029) Al-Karaji/Al-Karkhi — Introduction of higher positive integer exponents and negative exponents, recursive proofs of formulas that anticipate the modern concept of mathematical induction.

(965 - 1040) Al-Hazen — Experimental and theoretical research in optics and related mathematical issues.

(1048 - 1122) Khayyam — Graphical solutions of cubic equations using intersections of circles and other conics.

(1114 - 1185) Bhaskara — Extremely extensive and deep work on number theoretic questions including solutions to certain quadratic equations over the integers.

(1170 – 1250) Fibonacci — Introduction of Hindu-Arabic numeration to nonacademics, work on number theory including Fibonacci sequence, problems involving sequences of perfect squares in an arithmetic progression, Pythagorean triples.

(1201 – 1274) al-Tusi, Nasir — Early work on making trigonometry a subject in its own right.

(1219 – 1292) Bacon, Roger — Advocate for putting new mathematical discoveries to practical use.

(1220 – 1280) al-Maghribi — Commentaries on the apocryphal Books XIV and XV of Euclid's *Elements*.

(1225 – 1260) Jordanus — Limited use of letters, results on perfect versus nonperfect numbers.

(1285 – 1349) Ockham — Formulation of the concept of a limit, principle of expressing things as simply as possible (Ockham's razor).

(1313 – 1373) Heytesbury — Mean speed principle for uniformly accelerated motion.

(1323 – 1382) Oresme — Summations of certain infinite series, early ideas on the graphical representation of functions.

(1350 – 1425) Madhara — Infinite series formula for inverse tangent, formulas and estimates for $\pi$.

(1377 – 1446) Brunelleschi — First specifically mathematical study of drawing in geometric perspective.

(1380 – 1450) Al-Kashi — Free use of decimal fractions, trigonometry, accurate estimate for $\pi$.

(1401 – 1464) Cusa, Nicholas of — Early mention of cycloid curve, other contributions.

(1404 – 1472) Alberti — First written treatment of geometric perspective theory.

(1412 – 1492) Francesca — Most mathematical treatment of perspective during this time period.

(1412 – 1486) al-Qalasadi — Early versions of some modern notational conventions.

(1436 – 1476) Regiomontanus — Numerous translations of classical works, definitive account of trigonometry as a subject in its own right.

(1445 – 1500) Chuquet — Early versions of some modern notational conventions, "zillion" nomenclature for large numbers.

(1462 – 1498) Widman — First appearance of plus and minus signs.

(1465 – 1526) Pacioli — Comprehensive summary of mathematics at the time, published in print.

(1502 – 1578) Nunes — Mathematical theory of mapmaking.

(1512 – 1592) G. Mercator — Mathematical theory of mapmaking, important map projection with his name.