Math 153
Spring 2005
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Preparation suggestions for the final examination

Not finalized, but pretty close

The final examination will be about 65 per cent problems and 35 per cent historical or short answer. Some of the material will be from previous exams but a very substantial part will involve material not covered on the exams. For the historical part the main thing is to know the sequence of important developments and of important figures listed in the notes, basically in order and accurate to about a century or slightly less. As in the preceding exams, knowledge of what individuals did or clearly did not do (for example, Euler did not develop differential calculus) will be tested, and as on the second midterm exam it will be possible to earn full credit by answering most but not all parts of specific questions. A review summary is included at the end of this document.

For the mathematical part, the problems in the exercises are good practice material. Also, a thorough understanding of background material like elementary algebra and geometry, precalculus, and first year calculus will be assumed, and problems drawing upon such knowledge are likely to be on the examination. Some aspects have been mentioned explicitly in the course notes, but a few others may appear. Here are a few additional problems worth working as preparation:

1. It is important to be able to do simple inductive proofs of stated identities; for example, $1 + 3 + \cdots + (2n - 1) = n^2$.

2. It is also important to be able to work with trigonometric functions and polynomials at the precalculus level. The main identities to remember are $\sin^2 + \cos^2 = 1$ and the double angle formulas.

3. It is also important to be able to compute and reason with the long division property for positive integers: $a = bq + r$ where $0 \leq r < q$. — For example, there may be some questions involving the greatest common divisor of two numbers and the set of all linear combinations $sa + tb$ where s and t are integers, and you should know that if $a = bq + r$ with $0 < r < q$ then q is not a factor of a. There may be a problem on the Chinese Remainder Theorem or some step in working out examples.

4. Something related to Pell's Equation may appear (for example, see the archived exams).

5. Thre is likely to be something on cubic equations and their roots, possibly involving a step in the derivation of the cubic formula.

6. There may be something on polynomial equations satisfied by numbers of the form $\cos 360/n$ where n is a reasonably small positive integer (at most 15).

7. There may be something on continued fractions or Fibonacci numbers or arithmetic progressions.

8. There may be something on Diophantine equations at the level of the second midterm or the quizzes.

9. There may be something on the Pappus Centroid Theorem.

10. There may be something dealing with simple questions about infinite series at the level of first year calculus; for example, questions about evaluating infinite series like $\sum n2^{-n}$ or some series related to $\log_e 2$, $\log_e 10$ or $\pi/4$. [For example, if we know $\log_e 2$, use the identity $10 = 2^3 \cdot \frac{5}{4}$ and a series for $\log_e(5/4)$ to estimate $\log_e 10$ to a few decimal places.

11. There may be something dealing with simple differentiation of functions that were historically important at one time. For example, differentiating the Napier logarithm (see the course exercises and solutions) or expressing the derivative of $\mathrm{crd}\,\theta$ in the form $K\mathrm{crd}(a + b\theta)$ for suitable constants K, a, b.

Some of these items are related to problems which will appear on the exam (and others came from possible problems that were rejected).

Material covered on the examination

It will be easier to list the files in `http://math.ucr.edu/~res/math153` that definitely will NOT be covered on the final examination. All files that are not listed should be viewed as material that may be covered on this examination.

`braintest1.pdf`

`braintest2.pdf`

`braintest3.pdf`

`braintest4.pdf`
`history00b.pdf`
`history02d.pdf`
`history02f.pdf`
`history03e.pdf`
`history04b.pdf`
`history05b.pdf`
`history05c.pdf`
`history05d.pdf`
`history05f.pdf`
`history06a.pdf`
`history06e.pdf`
`history10.pdf`
`history11a.pdf`
`history13.pdf`
`history14b.pdf`
`impedance.pdf`
`morefermat.pdf`
`oldmagicsquare.pdf`
`oldmagicsquare2.pdf`
`polya.pdf`

```
transcurves.pdf
transcurves2.pdf
transcurves3.pdf
zzzevolution.pdf
```

It will not be necessary to study anything which is only in any of the subdirectories of the course directory.

Historical summary

(624 BCE - 548 BCE) Thales — First historic figure, results in geometry.

(580 BCE - 500 BCE) Pythagoras — Early and influential figure in development of mathematics, basic number-theoretic questions and some geometry (*e.g.*, Pythagorean Theorem).

(490 BCE - 430 BCE) Zeno — Formulated paradoxes which had a major impact on the subject.

(470 BCE - 410 BCE) Hippocrates of Chios — Computed areas, wrote early but lost books on mathematics.

(460 BCE - 400 BCE) Hippias — Quadratrix or trisectrix curve, good for trisection and circle squaring.

(428 BCE - 348 BCE) Plato — Influential ideas about how mathematics should be studied.

(417 BCE - 369 BCE) Theaetetus — Proof that all integral square roots of nonsquares are irrational.

(408 BCE - 335 BCE) Eudoxus — Proportion theory for irrationals, method of exhaustion to derive formulas.

(384 BCE - 322 BCE) Aristotle — Influential work on logic and its role in mathematics.

(380 BCE - 320 BCE) Menaechmus — Early work on conics, duplication of cube using intersecting parabolas.

(350 BCE - 290 BCE) Eudymus — Lost writings on the the history of Greek mathematics.

(325 BCE - 265 BCE) Euclid — Organized fundamental mathmatial material in the *Elements*, including material on geometry, number theory and irrational quantities.

(287 BCE - 212 BCE) Archimedes — Computations of areas and volumes, study of spiral curve, methods for expressing very large numbers.

(310 BCE - 230 BCE) Aristarchus — Heliocentric universe, astronomical measurements, simple continued fractions.

(280 BCE - 220 BCE) Conon — Associate of Archimedes also associated with the Archimedean spiral

(276 BCE - 197 BCE) Eratosthenes — Prime number sieve, earth measurements.

(262 BCE - 190 BCE) Apollonius — Extensive work on properties of conic sections, use of epicycles.

(190 BCE - 120 BCE) Hipparchus — Early work on trigonometry, use of latter in astronomy.

(190 BCE - 120 BCE) Hypsicles — Wrote Book XIV in *Elements*.

(10 AD - 75) Heron — Area of triangle expressed in terms of sides.

(70 - 130) Menelaus — Spherical geometry.

(85 - 165) Claudius Ptolemy — Trigonometric computations, astronomy.

(200 - 284) Diophantus — Equations over the integers and rational numbers, shorthand (syncopated) notation for expressing algebraic concepts.

(290 - 350) Pappus — Commentaries on earlier work and anthologies of such work, Centroid Theorem(s) for areas and volumes of surfaces and solids of revolution.

(335 - 395) Theon — Influential editing of the *Elements*, commentaries.

(410 - 485) Proclus — Commentaries on earlier Greek mathematics and its history.

(476 - 550) Aryabhatta — Base ten numbering system mentioned in his work, introduction of trigonometric sine function, more extensive and accurate tables of trigonometric functions.

(480 - 540) Eutocius — Commentaries publicizing the work of Archimedes.

(598 - 670) Brahmagupta — Base ten numbering system explicit, free use of negative and irrational numbers, zero concept included, work on quadratic number theoretic equations over the integers, some shorthand notation employed.

(790 - 850) al-Khwarizmi — Influential work on solving equations, mainly quadratics, beginning of algebra as a subject studied for its own sake.

(836 - 901) Thabit ibn Qurra — Original contribution to theory of amicable number pairs, extensive work translating Greek texts to Arabic.

(850 - 930) Abu Kamil — Further development of algebra.

(850 - 930) Al-Battani — Work in computational trigonometry and trigonometric identities.

(940 - 998) Abu'l-Wafa — Highly improved trigonometric computations, discussion of the mathematical theory or repeating geometric designs.

(950 - 1009) Ibn Yunus — Trigonometric computations and identities.

(953 - 1029) Al-Karaji/Al-Karkhi — Introduction of higher positive integer exponents and negative exponents, recursive proofs of formulas that anticipate the modern concept of mathematical induction.

(965 - 1040) Al-Hazen — Experimental and theoretical research in optics and related mathematical issues.

(1048 - 1122) Khayyam — Graphical solutions of cubic equations using intersections of circles and other conics.

(1114 - 1185) Bhaskara — Extremely extensive and deep work on number theoretic questions including solutions to certain quadratic equations over the integers.

(1170 – 1250) Fibonacci — Introduction of Hindu-Arabic numeration to nonacademics, work on number theory including Fibonacci sequence, problems involving sequences of perfect squares in an arithmetic progression, Pythagorean triples.

(1201 – 1274) al-Tusi, Nasir — Early work on making trigonometry a subject in its own right.

(1219 – 1292) Bacon, Roger — Advocate for putting new mathematical discoveries to practical use.

(1220 – 1280) al-Maghribi — Commentaries on the apocryphal Books XIV and XV of Euclid's *Elements*.

(1225 – 1260) Jordanus — Limited use of letters, results on perfect versus nonperfect numbers.

(1285 – 1349) Ockham — Formulation of the concept of a limit, principle of expressing things as simply as possible (Ockham's razor).

(1313 – 1373) Heytesbury — Mean speed principle for uniformly accelerated motion.

(1323 – 1382) Oresme — Summations of certain infinite series, early ideas on the graphical representation of functions.

(1350 – 1425) Madhava — Infinite series formula for inverse tangent.

(1377 – 1446) Brunelleschi — First specifically mathematical study of drawing in geometric perspective.

(1380 – 1450) al-Kashi — Free use of decimal fractions.

(1401 – 1464) Cusa, Nicholas of — Early mention of cycloid curve, other contributions.

(1404 – 1472) Alberti — First written treatment of geometric perspective theory.

(1412 – 1492) Francesca — Most mathematical treatment of perspective during this time period.

(1412 – 1486) al-Qalasadi — Early versions of some modern notational conventions.

(1436 – 1476) Regiomontanus — Numerous translations of classical works, definitive account of trigonometry as a subject in its own right.

(1445 – 1500) Chuquet — Early versions of some modern notational conventions, "zillion" nomenclature for large numbers.

(1462 – 1498) Widman — First appearance of plus and minus signs.

(1465 – 1526) Pacioli — Comprehensive summary of mathematics at the time, published in print.

(1502 – 1578) Nunes — Mathematical theory of mapmaking.

(1512 – 1592) G. Mercator — Mathematical theory of mapmaking, important map projection with his name.

(1465 – 1526) Ferro — Discovery of the cubic formula.

(1471 – 1528) Dürer — Research and writings on geometric perspective.

(1499 – 1545) Rudolff — Introduction of the radical sign $\sqrt{\ }$.

(1500 – 1557) Tartaglia — Independent derivation of cubic formula, extension to other cases.

(1501 – 1576) Cardan — Major work on algebra including cubic and quartic formula, phenomena involving complex numbers.

(1510 – 1558) Recorde — Introduction of an early form of the equality sign.

(1522 – 1565) Ferrari — Quartic formula for roots of a $^{\rm th}$ degree polynomial.

(1526 – 1573) Bombelli — Use of complex numbers, clarification of cubic formula in the so-called irreducible case.

(1540 – 1603) Viète — Major advances in symbolic notation including the use of letters for known and unknown quantities, results in the theory of equations, new insights into the properties of trigonometric functions and their identities, influential ideas and results about using algebraic methods to study geometric questions.

(1548 – 1620) Stevin — Popularization of decimals throughout Europe, work on centers of gravity, hydrostatics.

(1550 – 1617) Napier — Invention of logarithms.

(1552 – 1632) Bürgi — Independent invention of logarithms, findings published later than Napier and Briggs.

(1560 – 1621) Harriot — Introduction of symbolism in his works (modern inequality signs first appear her, inserted by editors).

(1561 – 1615) Roomen — Formulation of challenging algebraic problem solved by Viète.

(1561 – 1630) Briggs — Continued Napier's work and published tables of common base 10 logoritms.

(1564 – 1642) Galileo — Important examples of curves arising from moving objects, Galilean paradox regarding infinite sets.

(1571 – 1630) Kepler — Laws of planetary motion, use of infinitesimals to find areas, Wine Barrel Problem in maxima and minima.

(1574 – 1660) Oughtred — Invention of $\times$ for multiplication, invention of the slide rule.

(1577 – 1643) Guldin — Rediscovery of Pappus' Centroid Theorem.

(1584 – 1667) Saint-Vincent — Integral of $1/x$, refutation of Zeno's paradoxes using the concept of a covergent infinite series.

(1595 – 1632) Girard, Albert — Trigonometric notation, formula for area of a spherical triangle.

(1596 – 1650) Descartes — Refinements of Viète's symbolic notation including the use of x, y, z for unknowns, introduction of coordinate geometry in highly influential publication *Discours* *de la méthode*, but not including key features like rectangular coordinates or many of the standard formulas. The work on coordinate geometry was greatly influenced by classical Greek geometers such as Apollonius and Pappus and also by the work of Viète.

(1598 – 1647) Cavalieri — Investigations of areas and volumes, Cavalieri's cross section principle(s), integration of positive integer powers x^n by geometric means.

(1601 – 1665) Fermat — Important insights in number theory, coinventor of coordinate geometry (closer to the modern form than Descartes in many respects), preliminary work aimed at describing tangent lines and solving maximum and minimum problems. The work on coordinate geometry was greatly influenced by Apollonius in some respects and Viète in others.

(1602 – 1675) Roberval — Motion-based definition of tangents, numerous results on cycloids.

(1608 – 1647) Torricelli — Computations of integrals, results on cycloids, discovery of solid of revolution that is unbounded but has finite volume.

(1616 – 1703) Wallis — Free use of nonintegral exponents, extensive integral computations including x^r where r is not necessarily a positive integer, major shift to alebraic techniques for evaluating such integrals.

(1620 – 1687) Mercator, N — Standard infinite series for $\ln(1 + x)$.

(1622 – 1676) Rahn — First use of the standard division symbol $\div$.

(1623 – 1662) Pascal, Blaise — Many important contributions, including properties of cycloids and integration of $\sin x$.

(1628 – 1704) Hudde — Free use of letters to denote negative numbers, standard formulas for slopes of tangent lines to polynomial curves.

(1629 – 1695) Huygens — Numerous contributions, including solution of Galileo's isochone problem.

(1630 – 1677) Barrow — More refined definition of tangent line, realization that dirrerentiation and integration are inverse processes, integrals of some basic trigonometric functions.

(1633 – 1660) Heuraet — Mathematical description of arc length and computations for some important examples.

(1638 – 1675) Gregory, James — Integration of certain trigonometric functions, familiar power series for the inverse tangent, first attempt to write a textbook on advances leading to calculus.

(1640 – 1718) La Hire — Work on solic analytic geometry and other aspects of geometry.

(1643 – 1727) Newton — Coinventor of calculus. Formulation of earlier work in more general terms, recognition of wide range of applications, material eprxessed in algebraic as opposed to geometric terms. Main period of discovery in 1660s, publication much later. Work strongly linked to his study of physical problems, particularly planetary motion. His main work on the latter, *Principia*, was highly mathematical. He obtained the standard binomial series expansion for $(1 + x)^r$, where r is real. Notation for calculus included *fluxion* for derivative, *fluent* for integral and $\dot{x}$ for the derivative. Infinitesimals were not strongly emphasized, but the use of infinite series to express functions was stressed. Priority was placed on differentiation. Newton's applications of calculus was extremely important influence in determining the subsequent development of mathematics for well over a century.

(1646 – 1716) Leibniz — Coinventor of calculus. Formulation of earlier work in more general terms, recognition of wide range of applications, material eprxessed in algebraic as opposed to geometric Main period of discovery in 1670s, published in the next decade. Infinitesimals were strongly emphasized. The Leibniz notation, including dy/dx for derivative and $\int\ y\,dx$ for integral, became standard. Emphasis wason finding solutions that could be written in finite terms rather than infinite series. Priority was placed on integration. Leibniz also mad extremely important contributions to philosphy.

(1654 – 1705) Bernoulli, Jacob — Continued work on calculus and differential equations as well as many other important contributions.

(1661 – 1704) de L'Hospital — Publication of calculus book with formula bearing his name (purchased from Johann Bernoulli).

(1667 – 1748) Bernoulli, Johann — Continued work on calculus and differential equations as well as many other important contributions.

(1667 – 1748) de Moivre — Polar form of complex numbers $re^{i\theta} = \cos\theta + i\,\sin\theta$, also other important work.

(1685 – 1753) Berkeley — Extremely influential critique of infinitesimals in calculus ("ghosts of departed quantities").

(1685 – 1731) Taylor, Brook — Publication of series expansion and approximation formulas bearing his name.

(1698 – 1746) Maclaurin — Publication of previously known power series expansion bearing his name, geometrical studies, lengthy response to Berkeley phrased in classical geometric terms.

(1707 – 1783) Euler — Extremely important contributions to many areas of mathematics, including number theory, infinite series and solid analytic geometry.

(1713 – 1765) Clairaut — Development of solid analytic geometry, other contributions.

(1717 – 1783) d'Alembert — First suggestion of a concept of limit to circumvent logical problems with infinitesimals.

(1765 – 1802) Ruffini — First effort to prove that no quintic (5^{th} degree) formula exists.

(1777 – 1855) Gauss — Extremely important contributions to many areas of mathematics.

(1789 – 1857) Cauchy — Mathematical definition of limit in 1820 – nearly 150 years after the publication of Leibniz' work, also many other important contributions.

(1802 – 1831) Abel — Improved argument that radical formulas for roots of polynomials with degree ≥ 5 do not exist, insistence on a logically rigorous development of infinite series, other extremely important and far-reaching contributions over a very short lifetime.

(1815 – 1897) Weierstrass — The modern $\varepsilon - \delta$ definition of a limit, also many other important contributions.

(1831 – 1916) Dedekind — Mathematically rigorous description of the real number system, also many other important contributions.

(1845 – 1918) Cantor, Georg — Theory of infinite sets (the logical foundation of modern mathematics).

(1918 – 1974) Robinson, Abraham — Logically rigorous formulation of infinitesimals (nonstandard analysis)