More on sexagesimal conversion

Here is one more typical example.

PROBLEM. *Convert the ordinary fraction* 9/32 *to sexagesimal notation.*

SOLUTION. We have deliberately chosen a fraction whose denominator is divisible by a power of 60 so that the sexagesimal expansion will only have finitely many terms. In fact, $32 = 2^5$ divides 60^3 but does not divide 60^2 so we expect to have three terms in the expansion.

The first thing to do is to use the divisibility of 60^3 by 32 in order to write 9/32 as $a/60^3$ for some integer a:

$$\frac{9}{32} \quad = \quad \frac{a}{60^3} \quad \Longrightarrow \quad a \quad = \quad \frac{9 \cdot 60^3}{32} \quad = \quad \frac{3^2 \cdot 2^6 \cdot 3^3 \cdot 5^3}{2^5} \quad = \quad 60,750$$

In other words, we have

$$\frac{9}{32} \quad = \quad \frac{60,750}{60^3 = 216,000}$$

and to write this in sexagesimal form we need to write the numerator in the form

$$60,750 \quad = \quad A \cdot 60^2 \; + \; B \cdot 60 \; + \; C$$

where A, B, C are all integers between 0 and 59. We can find C by noting it is the remainder obtained from dividing 60,750 by 60:

$$60,750 \quad = \quad 1012 \cdot 60 \; + \; 30 \quad \Longrightarrow \quad C \quad = \quad 30$$

Similarly, B will be the remainder if we divide the integral quotient 1012 by 60:

$$1012 \quad = \quad 16 \cdot 60 \; + \; 52 \quad \Longrightarrow \quad B \quad = \quad 52$$

Finally, A will be the integtral quotient in the preceding expression:

$$A \quad = \quad 16$$

Thus the sexagesimal form of the fraction 9/32 is given by $16'\,52''\,30'''$ or $0;16,52,30$.

Truncated examples

The preceding construction yields exact sexagesimal expressions provided a real number is expressible as a fraction whose denominator is equal to 60^3 (equivalently, as a fraction whose denominator evenly divides 60^3. If this is not the case, then the usual Babylonian practice was to truncate the expression after three terms.

Example. Find the Babylonian expression for π.

We begin with a decimal expansion of π several decimal places:

$$\pi \quad = \quad 3.1415926535...$$

The first step is to find the largest fraction with denominator 60^3 which is less than $\pi - 3$. This fraction is equal to

$$\frac{[60^3(\pi - 3)]}{60^3}$$

where [...] denotes the greatest integer function, and since $60^3(\pi - 3)$ is equal to 30584.013... it follows that the approximaton to $\pi - 3$ is equal to

$$\frac{30584}{216000}$$

and we need to write this expression in Babylonian form. The first step is to divide 30584 by 60:

$$30584 \quad = \quad 509 \cdot 60 \ + \ 44$$

Next, divide the quotient 509 by 60:

$$509 \quad = \quad 8 \cdot 60 \ + \ 29$$

These imply that the Babylonian way of writing an approximation to π is $3; 8'29''44'''$ or $3; 8, 29, 44$.