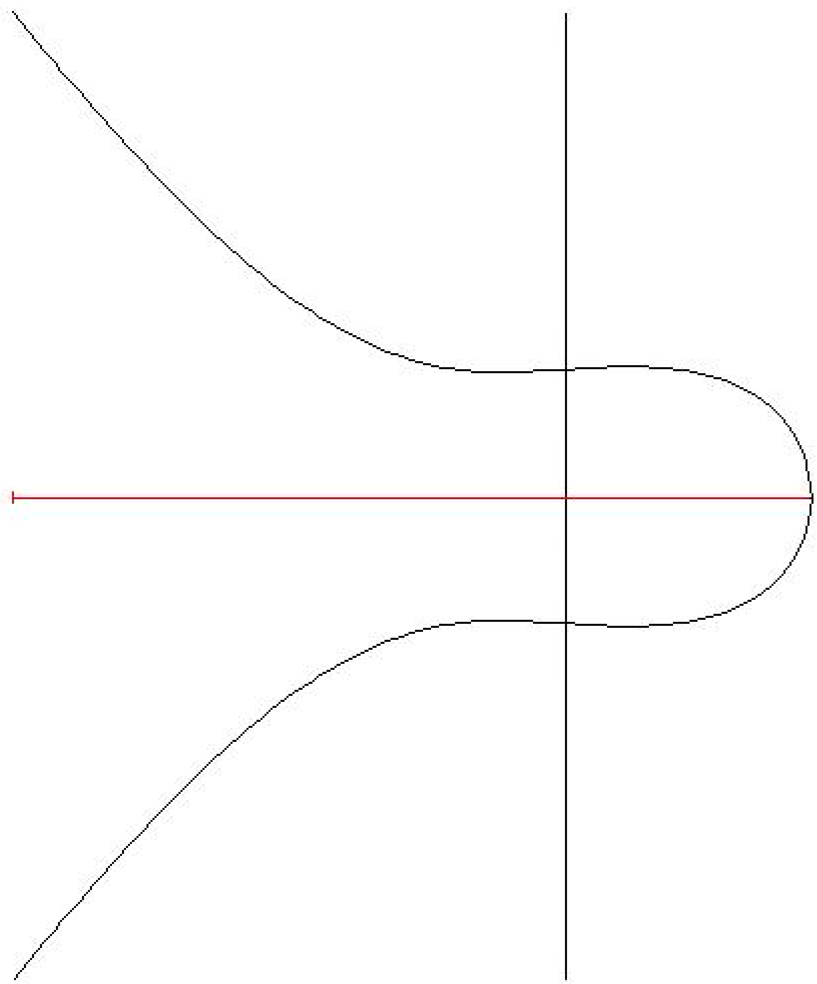
5. D. Graph for (5.C)

Here is the graph of the curve defined by $y(6-y) = x^3 - x$. The red horizontal line is defined by the equation $x = 3$, and the black vertical line is the y – axis. The curve meets the y – axis at the points $(6, 0)$ and $(0, 0)$.



This curve is an example of a particularly important class of objects known as **elliptic curves** (at least up to a change of variables). Such objects can be defined by equations of the form

$$y^2 = ax^3 + bx^2 + cx + d$$

in which the third degree term is nonzero and the cubic polynomial has no repeated roots. Numerous examples of such curves were studied by ancient Greek mathematicians, and beginning in the 17th century mathematicians began to study them systematically and discovered that they had many remarkable properties. Elliptic curves played a crucial role in the proof of Fermat's conjecture that the equations $a^n + b^n = c^n$ have no solutions in which all three variables are positive when $n > 2$, and they also have important applications outside of mathematics. Two easily stated examples are coding theory (cryptography) and certain topics in theoretical physics related to string theory. The following online PowerPoint files are excellent introductory sources for additional information:

www.math.brown.edu/~jhs/UbiquityOfEllipticCurves.ppt

http://www.google.com/url?sa=t&rct=j&q=elliptic%20curves%20string%20theory&source=web&cd=10&ved=0CGwQFjAJ&url=http%3A%2F%2Fgarsia.math.yorku.ca%2F%7Ezabrocki%2Fmath5020y0708%2Fhandouts%2FIntroEllipCurves.ppt&ei=J8qiT-2OCYSjiQKgl7jXBw&usg=AFQjCNF8gLbXa1buVD-Dx9Dx_clgoHYS5Q