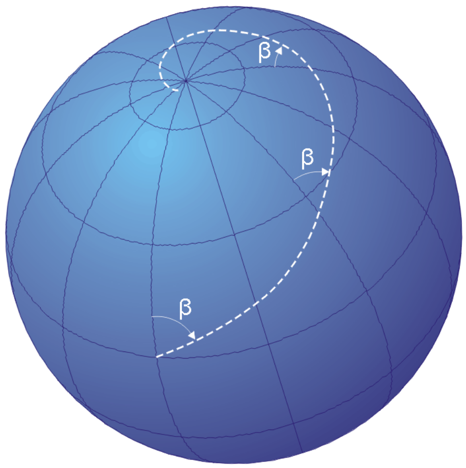
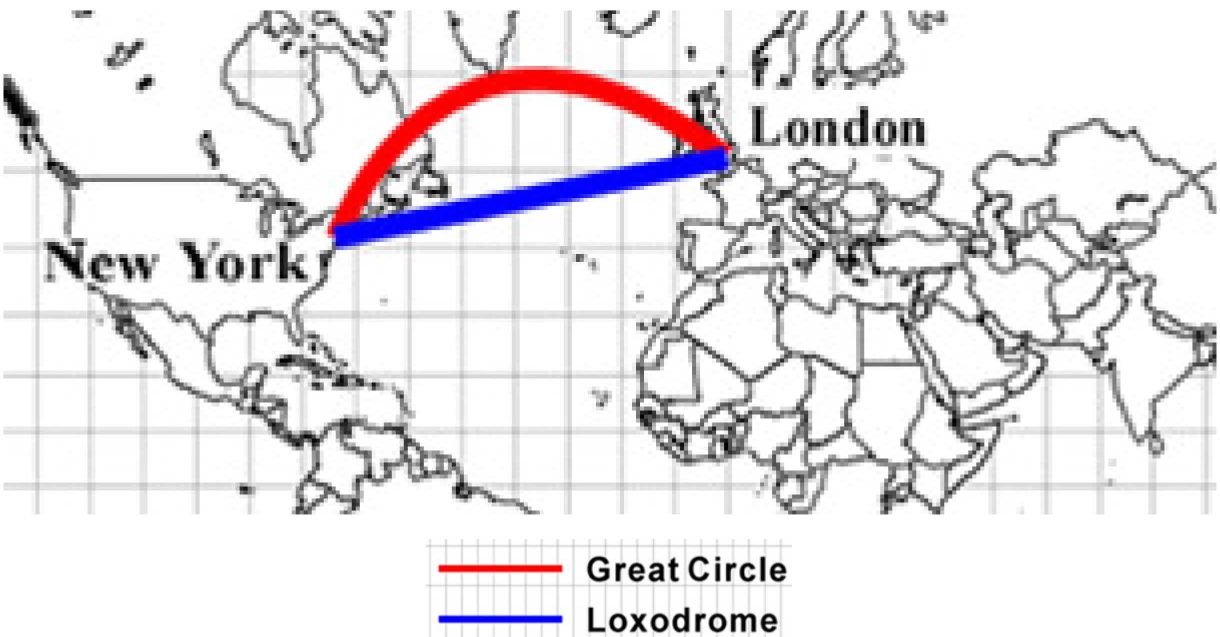
8.A. Loxodromes

We have noted that a loxodrome (or rhumb line) is a curve on the sphere for which the compass direction is constant and that under the Mercator projection a loxodrome corresponds to a straight line. The picture below shows a loxodrome from a point to the North Pole with a compass direction of β degrees measured clockwise from due north.



http://upload.wikimedia.org/wikipedia/commons/d/d6/Loxodrome.png

The shortest curve to the North Pole is along a longitude (or meridian), so in this case it is clear that the loxodrome is far from the shortest distance between two points on sphere in many cases. Here is a map showing the difference between paths from New York to London along a great circle route and a loxodrome:



(Source: http://www.ncgia.ucsb.edu/education/curricula/giscc/units/u014/figures/figure06.html)

Parametric equations for a loxodrome

As in the first picture of the preceding page, we shall let β denote the compass direction of a loxodrome, and we shall assume that the longitude at the starting point is equal to λ_0. If we let $m = \cot(\beta)$, then the following formulas describe a loxodrome parametrically in Cartesian coordinates as a function of the longitude λ:

$$x = r\cos(\lambda)/\cosh(m(\lambda - \lambda_0)),$$
$$y = r\sin(\lambda)/\cosh(m(\lambda - \lambda_0)),$$
$$z = r\tanh(m(\lambda - \lambda_0)).$$

See **http://en.wikipedia.org/wiki/Rhumb_line** for details of the derivation and further information.