

REVIEW SUGGESTIONS FOR THE FIRST EXAMINATION

The first midterm will be about 75 per cent mathematical problems and 25 per cent historical or short answer with opportunities for extra credit on the latter. The material coverage will go through the discussion of Apollonius' work on conics in `history4Y.pdf` and is likely to contain material from `history04a.pdf` and the first half of `history04b.pdf`.

MATHEMATICAL PROBLEMS. The old examinations give some idea of what to expect; these involve the use of algebra, precalculus or calculus to study issues related to the mathematics of ancient Egyptian, Babylonian and Greek civilizations. In addition to the problems from the old examinations, here are a few problems that were considered for the current examination but not included:

1. Show that the numbers 220 and 284 are amicable by explicit calculation.
2. Let $V(r)$ and $A(r)$ denote the volume and total surface area of a solid right circular cylinder whose radius is equal to r and whose height is equal to $2r$, so that a solid sphere of radius r can be inscribed in the cylinder. Prove that

$$\frac{d}{dr} V(r) = A(r).$$

3. Let C be the ellipse

$$\frac{x^2}{4} + y^2 = 1$$

and let d be a real number such that $0 < d < 2$. It is known that the slope of the tangent line to the ellipse at the point (p, q) has slope equal to $-p/4q$ if $q \neq 0$. For each d as above, prove that there is a unique $(p, q) \in C$ such that $q > 0$ and the line joining $(d, 0)$ to (p, q) is perpendicular to the tangent line to C at (p, q) . [*Hint:* Perpendicularity means that the slopes of the lines are negative reciprocals of each other. What is the slope of the line joining $(d, 0)$ to (p, q) ?

HISTORICAL QUESTIONS. These will involve major mathematical contributions or advances in various cultures, the achievements of individuals and their best known works, and some basic knowledge of the time sequence of important developments or important figures (say within a generation or so). Some true/false type questions may be more difficult than they seem, and one important thing to recognize is that the format will be somewhat different from the older exams (although the substance will be similar). A few things worthy of particular attention are the summaries of Plato's views on mathematical practice, the contents of Euclid's *Elements*, and ways in which the latter did not meet current standards for logical completeness.

Some relevant files. Obviously the files `history*.pdf` for Units 0 – 4.Y contain material for the examination as do the exercise and solution files for the first four units. In addition there are the following:

- Unit 1: `base60change.pdf`, `degreeconversions.pdf`, `frustrum.pdf`.
- Unit 2: `circleright.pdf`, `circleright2.pdf`, `semicircle.pdf`, `vectorproofs.pdf`.
- Unit 3: `bisection.pdf`, `euclids-prime-proof.pdf`, `historical-maps1.pdf`, `pythagorean-thm.pdf`.
- Unit 4: `ellipse-reflection.pdf`, `spiralregions.pdf`.