Math 205A
Fall 2014

UPDATED GENERAL INFORMATION — OCTOBER 6, 2014

Assignments for Unit **I**

Working the exercises listed below is **strongly recommended.**

The following exercises are taken from Munkres:

- Munkres, Section 7: 4
- Munkres, Section 9: 5

The following references are to the file file `gentopexercises2014.pdf` in the course directory.

- Additional exercises for Section I.2: 3
- Additional exercises for Section I.4: 1

Reading assignments from solutions to exercises

Another strong recommendation is to read through the solutions to the following problems in the file `math205Asolutions01.pdf`; some additional comments on the significance of these exercises are also given below.

- Additional exercises for Section I.1: 1
- Additional exercises for Section I.2: 1, 2, 4
- Additional exercises for Section I.3: 1, 3
- Additional exercises for Section I.4: 3, 4

Comments

This unit is mainly a review of material that was probably covered in prerequisite courses at the undergraduate level, but the treatement of transfinite cardinals in the notes and exercises might go beyond what was contained in such courses. A second major goal of the unit is to introduce notation that is probably somewhat different from the conventions typically seen in undergraduate courses; some are in more common use than others, but all of them appear outside of these notes.

Here are some references to

`http://math.ucr.edu/ res/math145A-2014/set-theory-notes.pdf` for the various sections of the notes. The sections of Unit I are not entirely in logical order, and a better logical sequence would be **I.2** $\rightarrow$ **I.4** $\rightarrow$ **I.3** $\rightarrow$ **I.1**, so we shall give references to sections in that order. The main point of Section I.2 is to introduce the notational conventions mentioned above and to stress some aspects of set theory which are particularly important for this course but are not generally covered in much detail at the undergraduate level. The latter applies particularly to the exercises for Section I.2 listed above (for solving and for reading). In connection with the notation for images and inverse images, we should give an example of a set A and a subset B such that $B \subset A$ and $B \in A$; the easiest such example is to take $A = \{\emptyset\}$ and $B = \emptyset$. The appropriate references to `sets-theory-notes.pdf` are Sections II.2–3, III.1–3, and all of Unit IV. The main reference for Section I.4 is Unit V of `set-theory-notes.pdf`. Section I.3 discusses transfinite cardinals fairly extensively, with emphasis on the most basic results like the difference between the cardinality of the integers and the cardinality of the real numbers. The relevant sections of `set-theory-notes.pdf` are Sections VI.1–4. Although the Axiom of Choice is mentioned in Section I.3, it is not really needed for the results in that section. Howver, at some point one does need the Axiom of Choice when dealing with transfinite cardinals; this result is mentioned in Section I.1 of the course notes, with detailed coverage in Sections VI.6 and VII.1–4 of `set-theory-notes.pdf`. Later in the course some of these results will be used, but for the time being there is no need to know much if anything about the formal axiom; one needs this assumption to justify statements like "choose some $a \in A$ such that ... " and for the purposes of this course it is usually enough to agree that one can make such non-constructive choices.

There is another general issue involving set theory which could have been mentioned in Section I.1. Early in the 20^{th} century, one concern about set theory involved the possibility that a set could be a member of itself (Is $A \in A$ ever true?). Intuitively it probably seems clear that something like this should not happen, and Section III.4 of `set-theory-notes.pdf` discusses standard ways of avoiding potential examples with such properties.

Finally, here are some more specific remarks on the motivation for listing a few of the exercises. Beginning with Unit V, we shall be interested in some constructions involving an object with topological structure and an equivalence relation on that object. These will involve equivalence relations which are often described indirectly, and Additional Exercise I.2.3 provides a framework for doing so in many cases. We have included Additional Exercise I.1.1 because it is an example of an argument which is somewhat longer than those appearing in many undergraduate course, and Additional Exercise 1.2.1 contains typical examples of proofs that two sets are equal; before reading through the solution, it might be helpful to draw some pictures in the plane when the various subsets are intervals in the real line. Additional Exercise I.3.1 is a fact which is used at various points throughout the course, and Additional Exercise I.3.3 (Tukey's Lemma) is an abstract version of several results in other areas of mathematics, including the fact that every vector space has a basis. Finally, Additional Exercises I.4.3–4 are included to review some basic facts about convergent sequences (for I.4.3) and the uses of least upper bounds and greatest lower bounds to prove statements involving real numbes (for I.4.4).