Math 205A
Fall 2014

UPDATED GENERAL INFORMATION — OCTOBER 22, 2014

The first in-class examination

The first in-class examination, which will take place on **Wednesday, October 29**, will cover everything through Section 5.1 in `gentop-notes.pdf`).

The problems on the exam will be either proofs of results in the notes or similar to the easy and moderately challenging exercises which were strongly recommended in earlier postings. Here are a few sample questions to consider. Some might be more demanding than the problems which will appear on the exam but not dramatically so.

1. (i) Prove that if X and Y are topological spaces with the associated indiscrete topologies, then the product topology on $X \times Y$ is also the indiscrete topology.

(ii) Suppose that X and Y are two spaces with the associated finitary topologies. Under what conditions is the product on $X \times Y$ the finitary topology on this set, and under what conditions is it not the finitary topology?

2. Suppose that X is a compact metric space with metric d. Prove that there are two points $a, b \in X$ so that $d(a,b) \geq d(u,v)$ for all $u, v \in X$.

3. For each of the statements below, indicate whether it is true or false and give brief reasons for your answer.

(i) If X is connected, then every open subset of X is connected.

(ii) If X is locally connected, then every open subset of X is locally connected.

(iii) The cardinality of the set of arcwise connected subsets in the coordinate plane equals the cardinality of the set of connected subsets in the plane.

(iv) If $A \subset X$ is closed, then every connected component of A is closed in X.

(v) If $A \subset X$ is closed, then every arc component of A is closed in X.

4. Let $X \subset \mathbb{R}^n - \{\mathbf{0}\}$ be compact. Define the cone on X to be the quotient $Cone(X)$ of $X \times [0,1]$ by the equivalence relation $\mathcal{R}$ whose equivalence classes are the one point sets $\{(x,t)\}$ where $t > 0$ and the slice $X \times \{0\}$. Let $\pi : X \to Cone(X)$ be the quotient projection, and take the quotient topology on the cone.

(i) Show that if $f_0 : X \to \mathbb{R}^{n+1} \cong \mathbb{R}^n \times \mathbb{R}$ is defined by $f_0(x,t) = (tx, t)$, then there is a unique continuous mappint $f : Cone(X) \to \mathbb{R}^{n+1}$ such that $f_0 = f \circ \pi$.

(ii) Prove that f maps $Cone(X)$ homeomorphically onto its image, showing in particular that $Cone(X)$ is a Hausdorff space. [*Hint:* Why is it enough to show that f is 1–1?]

5. (i) Explain why the polynomial function

$$p(x,y) \;=\; 6x^5 \;-\; 5x^4y \;+\; 4x^3y^3 \;-\; 3x^2y^3 \;-\; 2x^4y \;+\; y^5$$

takes the value $1/\sqrt{2}$ at some point (x_0, y_0) in the solid square $[0,1] \times [0,1]$.

(ii) Let f be a continuous nonnegative real valued function on $\mathbb{R}^2$ such that

$$\lim_{(x,y)\to\infty} f(x,y) \quad = \quad 0\ .$$

Prove that f has a maximum value at some point of $\mathbb{R}^2$. Give an example to show that f does not necessarily have a minimum value.