Math 205A
Fall 2014

UPDATED GENERAL INFORMATION — OCTOBER 24, 2014

Added files for Unit **V**

Three files have been added to the course directory. Two of them, `moebius.pdf` and `moebius2.pdf`, fill in the details for a construction which was outlined in the lectures; namely, showing that the quotient space construction for the Möbius strip

$$M \quad = \quad [0,1] \times [-\tfrac{1}{3}, \tfrac{1}{3}]/\mathcal{R}$$

where $\mathcal{R}$ is the equivalence relation generated by $(0,t) \sim (1,-t)$, is homeomorphic to a subspace of $\mathbb{R}^3$ which looks like the standard picture of this space. The first document defines an explicit continuous mapping from M to $\mathbb{R}^3$ which is 1–1; by Theorem III.1.8 this map must map M homeomorphically to its image because M is compact (it is the continuous image of a compact space) and $\mathbb{R}^3$ is Hausdorff. The second document contains a verification that the mapping from M to $\mathbb{R}^3$ is in fact 1–1.

The third document, `scissors+paste.pdf`, discusses a problem which generalizes the Scissors and Paste Problem in Section V.2. For the sake of convenience, we recall that if $\{X_\alpha \mid \alpha \in A\}$ is an indexed family of spaces and $Y = \cup_\alpha X_\alpha$ then the disjoint union space

$$\coprod_\alpha X_\alpha$$

is the set of all $(p,\alpha) \in Y \times A$ such that $p \in X_\alpha$, and the topology on this space is given by all subsets $\coprod_\alpha U_\alpha$ such that U_α is open in X_α for every $\alpha \in A$. It follows that each $X_\alpha \times \{\alpha\}$ is an open and closed subset of the disjoint union, and this subset is homeomorphic to X_α via the map sending (p,α) to p.

Here is the statement considered in `scissors+paste.pdf`:

> *Let $\mathcal{U} = \{U_\alpha \mid \alpha \in A\}$ be an open covering of a topological space X, and define an equivalence relation $\mathcal{R}$ on the disjoint union space $\coprod_\alpha U_\alpha$ such that $(x,\alpha)\ \mathcal{R}\ (y,\beta)$ if and only if $x = y$ and this point lies in $U_\alpha \cap U_\beta$. Then the quotient space*
>
> $$\left(\coprod_\alpha U_\alpha\right)/\mathcal{R}$$
>
> *is homeomorphic to X.*

As indicated in the lectures, none of this material will be covered on the first in-class examination.