Math 205A
Fall 2014

UPDATED GENERAL INFORMATION — NOVEMBER ??, 2014

Assignments for Unit **VII**

Working the exercises listed below is **strongly recommended.**

The following exercises are taken from Munkres:

- Munkres, Section 51: 2, 3
- Munkres, Section 52: 1*a*
- Munkres, Section 58: 1, 6

The following exercises are from Hatcher:

- Hatcher, pp. 18 – 20: 5, 12

The following references are to the file `fundgpexercises2014.pdf` in the course directory.

- Additional exercises for Section VII.0: 1, 2, 3*abc*, 4, 6
- Additional exercises for Section VII.1: 2, 3, 4
- Additional exercises for Section VII.2: 2
- Additional exercises for Section VII.3: 1, 2, 5
- Additional exercises for Section VII.4: 1, 4

Reading assignments from solutions to exercises

Another strong recommendation is to read through the solutions to the following problems in the files `math205Asolutions07.pdf`; some additional comments on the significance of these exercises are also given below.

- Additional exercises for Section VII.3: 3, 6
- Additional exercises for Section VII.4: 2, 3

Comments

This is the beginning of the second part of the course. It will be convenient to use some language from category theory at many points, so there is a preliminary section (VII.0) on this topic which has no counterpart in Munkres. Roughly speaking, category is a framework for studying abstract mathemtical systems together with "good" functions or morphisms from one example to another. Vector spaces and linear transformations are one simple example, as are sets and functions from one set to another as defined in this course. The concepts from category theory which will be needed are summarized on pages $vi - vii$ of `fundgp-notes.pdf`, and the added file `functors+isomorphisms.pdf` contains all the details for a proof which was given in the lectures for covariant functors: *If F is either a contravariant or a covariant functor from one category $\mathcal{A}$ to another category $\mathcal{B}$ and $g : X \to Y$ is an isomorphism in $\mathcal{A}$, then $F(g)$ is an isomorphism in $\mathcal{B}$.*

On a less formal and more geometrical side, the main concept in the second part of this course is the notion of **homotopy**, which can be thought of as a 1-parameter deformation or perturbation of some geometrical entity such as a continuous function.

Reading assignments. As usual, the solutions assigned for reading are meant to illustrate certain points. Additional Exercise VII.3.3 is a proof that the fundamental group of a product is a product of the fundamental groups of the factors, and Additional Exercise VII.3.6 shows that the Cantor set is not homotopy equivalent to an open subset in some $\mathbb{R}^n$. Furthermore, Additional Exercise VII.4.2 describes a simple case of a construction which arises which plays an important role in topology, and Additional Exercise VII.4.3 is an application of the universal mapping property for products, which is discussed in `characterizations.pdf`.