UPDATED GENERAL INFORMATION — NOVEMBER ??, 2014

Assignments for Unit **VIII**

Working the exercises listed below is **strongly recommended.**

The following exercises are taken from Munkres:

- Munkres, Section 53: 1, 2, 5, 6
- Munkres, Section 58: $2acdfghij$ (see the note in `fundgpexercises2014.pdf`), $9bcde$ with the definition of degree given in `fundgpexercises2014.pdf`
- Munkres, Section 59: 2, 4

The following exercises are from Hatcher:

- Hatcher, pp. 38 – 40: 3, 10, 13, $16ac$, 17 with $S^1 \times S^1$ replacing $S^1 \vee S^1$
- Hatcher, pp. 78 – 82: 2

The following references are to the file `fundgpexercises2014.pdf` in the course directory.

- Additional exercises for Section VIII.3: 1, 3, 4, 5
- Additional exercises for Section VIII.4: 1, 2, 3

Reading assignments from solutions to exercises

Another strong recommendation is to read through the solutions to the following problems in the files `math205Asolutions08.pdf`; some additional comments on the significance of these exercises are also given below.

Munkres, Section 53: 4

Munkres, Section 54: 7

- Additional exercises for Section VIII.1: 1

Comments

This unit introduces the two main concepts in Part II of Munkres: The *fundamental group* of (pointed) space is an object which gives an algebraic "picture" of the closed curves in the space. The study of this group is closely tied to the study of *covering spaces*. One motivation for the latter arises from issues involving the definition of the angle polar coordinate θ in the plane. At each point other than the origin, one can find a neighborhood on which a continuous version of θ can be defined, but experience shows that we cannot do this continuously for all nonzero points in the plane. Instead, the local definitions near the various points can be viewed as pieces of some multiple valued continuous function on $\mathbb{R}^2 - \{\mathbf{0}\}$. A covering space of the latter is a topological space X with a continuous mapping p into $\mathbb{R}^2 - \{\mathbf{0}\}$; one property of p is that each point $x \in X$ has an open neighborhood which maps homeomorphically to an open neighborhood of $p(x)$, and one can view multivalued continuous functions into $\mathbb{R}^2 - \{\mathbf{0}\}$ as continuous functions into some covering space $X \to \mathbb{R}^2 - \{\mathbf{0}\}$. For the standard example of the angle function θ, one can take X to be $(0, \infty) \times \mathbb{R}$ and $p(r, \theta)$ to be the familiar polar-to-rectangular change of coordinates formula $(r \cos \theta, r \sin \theta)$.

Reading assignments. Exercise 54.7 in Munkres is closedley related to Additional Exercise VIII.1.1, which explains the reason for the subscript "1" in the notation $\pi_1(X, x_0)$. The latter suggests that $\pi_1(X, x_0)$ is the first group in a sequence, and the exercise explains how one can define the higher terms $\pi_n(X, x_0)$ of this sequence (where $n \geq 2$), at least for a special but important class of spaces; everything can be done in much greater generality if one adopts a metric-free approach to topologizing function spaces as in Munkres.

In a much different direction, Exercise 53.4 from Munkres is needed in the computation of the fundamental group for the Klein bottle, which is a nonabelian group which has no elements of finite order and an index 2 (normal) subgroup isomorphic to $\mathbb{Z} \times \mathbb{Z}$.