Math 205A
Fall 2014

UPDATED GENERAL INFORMATION — NOVEMBER ??, 2014

Assignments for Unit **IX**

Working the exercises listed below is **strongly recommended.**

The following exercises are taken from Munkres:

- Munkres, Section 68: 2, 3
- Munkres, Section 69: 1, 3, 4
- Munkres, Section 70: 3

The following references are to the file `fundgpexercises2014.pdf` in the course directory.

- Additional exercises for Section IX.1: 1, 4
- Additional exercises for Section IX.2: 2
- Additional exercises for Section IX.3: 1, 2, 4
- Additional exercises for Section IX.4: 1, 2

Reading assignments from solutions to exercises

Another strong recommendation is to read through the solutions to the following problems in the files `math205Asolutions09.pdf`; some additional comments on the significance of these exercises are also given below.

Munkres, Section 70: 1

- Additional exercises for Section IX.1: 2, 3
- Additional exercises for Section IX.2: 1, 3
- Additional exercises for Section IX.3: 3, 5
- Additional exercises for Section IX.4: 3

Comments

The usefulness of the fundamental group depends upon having effective tools for computing it for specific examples, and Unit IX is devoted to describing some of these tools. There is a considerable amount of group theory that must be developed; the standard entry level courses in algebra develop significant parts of group theory, but the emphasis is more on finite groups, and in geometry and topology infinite groups are extremely important. These tools are also highly relevant to the question of realizing abstract groups as fundamental groups of spaces which have good topological or geometrical properties.

The central result of this unit (the Seifert-van Kampen Theorem) shows that, in many cases, we can compute the fundamental groups of spaces built out of known pieces if we know the fundamental groups of these pieces and how the groups are algebraically related to each other.

One important theme in the group-theoretic discussions is that certain important types of groups can be described by axioms known as *Universal Mapping Properties*. In particular, this is true for some basic group-theoretic constructions such as free groups, free products of groups, and pushouts of certain systems of group homomorphisms.

Reading assignments. Additional Exercise IX.1.2 is an example of how the Universal Mapping Property for free groups can be applied, and Additional Exercise IX.1.3 establishes an important and probably not obvious relationship between the autormorphism group of a free group and the autormorphism group of the corresponding free abelian group. Applications of the Universal Mapping Property for free products of groups are given in Additional Exercise IX.2.1, and Additional Exercise IX.2.3 is an example showing how an automorphism of a free group can differ from the corresponding automorphism of a free abelian group (this is closely related to Additional Exercise IX.1.3, which was mentioned in the preceding sentence). Additional Exercises IX.3.3 and IX.3.5 are related to typical applications of the Seifert-van Kampen Theorem. Finally, the goal of Additional Exercise IX.4.3 is to construct a space whose fundamental group is isomorphic to the additive group of rational numbers.

IMPORTANT. Problems like Additional Exercises IX.1.2, IX.2.1, IX.3.3 and IX.3.5 have appeared on course and qualifying exams in the past and are are likely to do so in the future, so the solutions to these exercises should be particularly well understood.

On the other hand, it is enough to understand the solutions to the remaining reading assignments from the first two sections of this unit, and it is enough to be aware of the main conclusion in Additional Exercise IX.4.3, whose solutions requires the introduction of some new approaches which do not appear elsewhere in this course.

Added documents for Unit **VII**

Two files have been added to provide further information about Example 2 on page 362 of Munkres. The first is `def-retr-munkres362.pdf`, and it describes the construction of a strong deformation retraction for the inclusion of a Figure Eight space into $\mathbb{R}^2$ with two points removed. The treatment in this document reflects the construction in the lectures, which is not quite the same as the construction in Munkres. The second document is `def-retr-munkres362a.pdf`; it describes the mappings in question more explicitly and indicates how one can verify that these maps define retractions and homotopies.