I.2

Awkward notational points

A Ordered pairs vs. open intervals.

Same notation (a, b)

Munkres uses $a \times b$ for ordered pairs, but

1. this has its own disadvantages,
2. it is rarely if ever seen elsewhere.

B Values of functions, images & inverse images of subsets.

What if $x \in X$ & $x \subseteq X$? Example $X = \{\emptyset\}$.

See Kelley Gen. Topology

$f(x)$ = value of function $f : X \to Y$.

$f[A]$ = image of A under f, $f^{-1}[B]$ = inverse image of B w.r.t. f *

* definable even if there is no inverse function for f. – Also see Exercise I.2.2.

C Synonyms for functions that are 1-1 or onto

D Definitions of function $f : X \to Y$

graph $\subseteq X \times Y$, X = domain <u>AND</u> Y = codomain

$f(x) = g(x) = \frac{1}{x^2+1}$ $f : \mathbb{R} \to \mathbb{R}$, $g : \mathbb{R} \to [0,1]$

DIFFERENT.

Not that crucial in Part One, but unavoidable in Part Two.

Sum or disjoint union

$$A \amalg B = A \times \{1\} \cup B \times \{2\}$$

See also disjoint-union.pdf

I.3

From time to time we shall need some basic facts about transfinite cardinal numbers, but on the level of $|\mathbb{Z} \times \mathbb{Z}| = |\mathbb{Z}|$, $|\mathbb{R} \times \mathbb{R}| = |\mathbb{R}|$, $|\mathbb{R}| = 2^{|\mathbb{Z}|}$.

I.4

We shall need the fact that $\mathbb{R}$ is a complete ordered field, and the consequence that $\mathbb{Q}$ is dense in $\mathbb{R}$. We shall also use material from undergraduate real variables courses freely. For example, the existence of a positive square root for $a > 0$.