III.1

Only some background before switching from light to normal coverage

Want to prove:

$f: [a,b] \longrightarrow \mathbb{R}$ cont. $\Rightarrow$ takes $\begin{Bmatrix} \max \\ \min \end{Bmatrix}$ values.

Standard approach — Munkres, Thm 27.1

Heine-Borel (-Lebesgue) Thm. ABSTRACT VERSION

X linearly ordered set such that every nonempty subset has $\begin{matrix} \text{L.U.B.} \\ \text{G.L.B.} \end{matrix}$; put the order topology on X (gen by sets $\{x < x_0\}$, $\{x > x_1\}$)

Then if we have open sets U_x cont. $x \in X$ $(\forall x)$, then $\exists$ finite subcollection $U_{x_1}, \ldots, U_{x_n}$ s.t. $U_{x_1} \cup \ldots \cup U_{x_n} = X$.

X is compact

Prime example $X = [a,b] \subseteq \mathbb{R}$.

We now shift to normal coverage