The Cantor set

The *Cantor set* was mentioned explicitly in Section I.4, which summarizes basic facts about the real number system (Section I.4). There are two ways of describing this set. The approach in Munkres, which is the standard one in most texts, is to view the set as the result of removing various subsets from the unit interval. Here is a drawing which shows the first few steps in the construction, which begins with the unit interval and proceeds by successive removal of the middle thirds of the intervals remaining after the preceding step:



(Source: http://library.thinkquest.org/2647/chaos/cantor.htm)

The **YouTube** video <u>http://www.youtube.com/watch?v=Ni4NXSy17il</u> contains an animation of this process. A second approach, which is more elementary in some respects but less useful in others, is to describe the set in terms of the 3 – adic expansion of points in the unit interval as infinite sums of terms $a_k/3^k$, where k runs through all positive integers and each a_k is 1, 2 or 3. From this viewpoint, the points of the Cantor set are those which have an expansion with each $a_k = 0$ or 2. Note that some points have one expansion of this type but another which is not of this type; e.g., 1/3 has the 3 – adic expansions 0.1 and 0.0222222....

Finally, the *Wikipedia* article <u>http://en.wikipedia.org/wiki/Cantor set</u> is one reference for further information about the Cantor set.