Addendum to the solution for Problem 4

One can also use fundamental groups to prove that T^n and T^m are not even homotopy equivalent if $m \neq n$. More generally, if X and Y are arcwise connected spaces with $x \in X$ and $y \in Y$, then $\pi_1(X, x)$ and $\pi_1(Y, y)$ are isomorphic if X and Y are homotopy equivalent. For a **homotopy equivalence of spaces with base points** from (X, x) to (Y, y), this is an immediate consequence of the the fact that the fundamental group is a covariant functor from spaces with base points to groups. However, one still has an isomorphism of fundamental groups even if there is a homotopy equivalence in the category of spaces, with no assumptions that the mappings or homotopies are base point preserving.

One reference for this fact is Munkres, Theorem 58.7, on pages 364–365. An important step in proving this result is Lemma 58.4, which is on pages 363–364. The latter compares the maps of fundamental groups induced by two mappings $f, g : X \to Y$ which are homotopic but not necessarily basepoint preservingly homotopic; in fact, the lemma does not even include an assumption that both maps send a base point in X to the same point in Y.