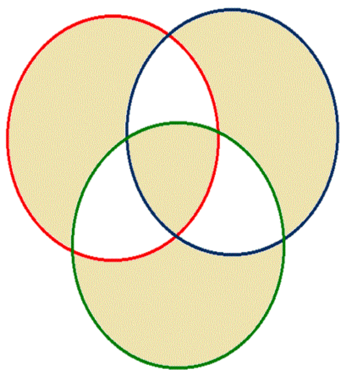
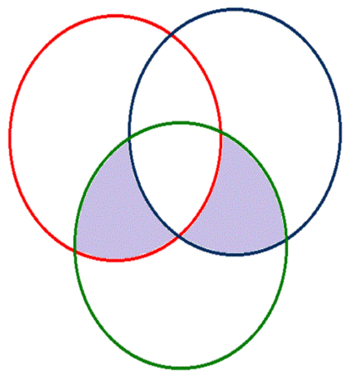
Drawing to accompany Additional Exercise I.1.1

Since the hint for this exercise suggested drawing Venn diagrams, here are two such drawings. The first (on the left) illustrates the associativity law $(A \oplus B) \oplus C = A \oplus (B \oplus C)$ and the second one illustrates the distributive law $A \cap (B \oplus C) = (A \cap B) \oplus (A \cap C)$. In each of these drawings, the circles at the top represent A and B while circle at the bottom represents C, and one shows that the shaded regions depict both the set on the left and the set on the right of the asserted equations.





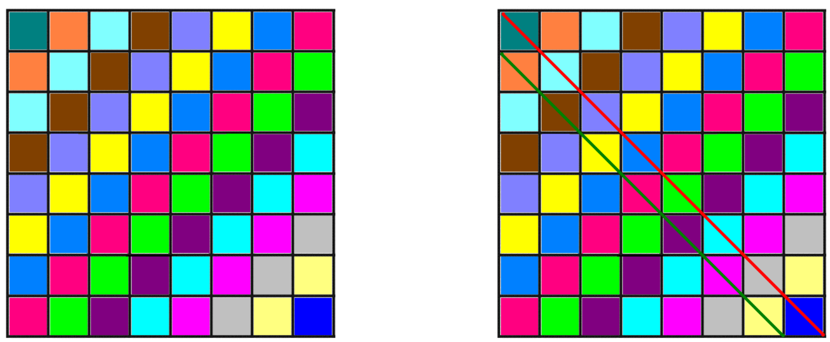
Note. If one defines an abstract Boolean algebra to be a set A with abstract union, intersection and complementation operations which satisfy the standard set – theoretic identities as in **http://en.wikipedia.org/wiki/Boolean_algebra_%28structure%29**, then one can define $\oplus$ exactly as in the exercise, and it satisfies the properties derived in the exercise. This fact has far – reaching implications. For example, it is central to the proof that every abstract Boolean algebra A is isomorphic to a subalgebra of the Boolean algebra $\mathcal{P}(X)$ of subsets of some set X. This result is due to M. H. Stone; here is an online reference for additional information (it uses concepts developed later in this course):

http://en.wikipedia.org/wiki/Stone%27s_representation_theorem_for_Boolean_algebras

Incidentally, despite the apparently abstract nature of the result, the motivation for proving it arose in connection with some sophisticated questions in functional analysis.

Drawing to accompany Additional Exercise I.2.4

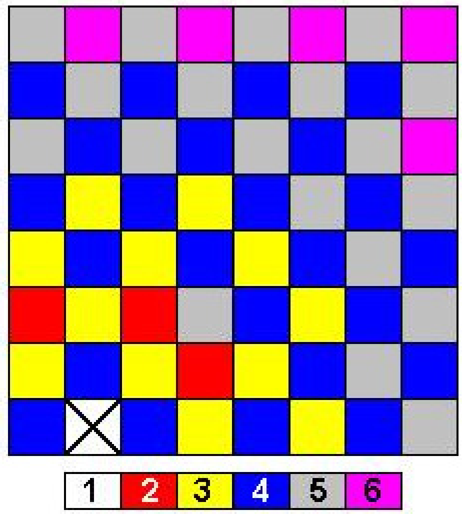
Assume we label the squares by two positive integers on the board from left to right and from the bottom to the top. The first step in the argument is to show that a bishop can move diagonally to another square of the same color. In other words, if the bishop is located at the point whose horizontal coordinate is i and whose vertical coordinate is j, then the bishop can move one square up or down, to the square whose horizontal coordinate is $i+1$ and whose vertical coordinate is $j-1$, or to the square whose horizontal coordinate is $i-1$ and whose vertical coordinate is $j+1$, provided there are such squares on the board. Thus each diagonal lies in an equivalence class of points such that a bishop can move from one square to another in the class, and since there are exactly 15 diagonals in the drawing, this means there are at most 15 equivalence classes (see the drawing on the left).



The final step in the argument is to note that the points on the red and green lines in the right hand drawing also lie in the same equivalence class. Since the two lines contain exactly one square of each color, it follows that there are at most two equivalence classes of squares, and they are distinguished by whether $i+j$ is even or odd. In fact, there are exactly two such equivalence classes, for if the bishop moves one square from the position with coordinates i and j to a square with coordinates p and q, then by construction the sums $i+j$ and $p+q$ are both even or both odd.

See the next page for remarks on the squares that a knight on a chessboard can reach.

Knight move illustrations



***Explanation*:** The color code indicates when a knight will get from the original position marked with an **X** to a given square on the chessboard. For example, the possible positions after one move are colored red, and the possible positions after two moves are colored yellow. Note that the knight can reach every square in at most five moves. It also follows that if the knight starts at an arbitrary square, then it can reach any other square within **10** moves.