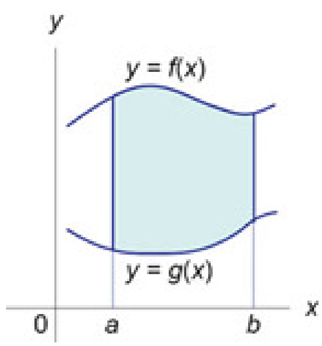
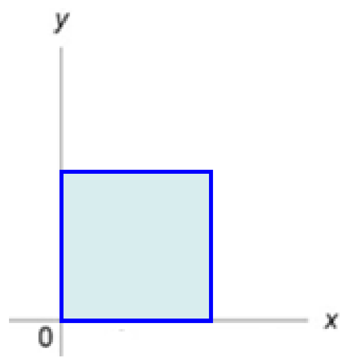
Drawing to accompany Additional Exercise II.4.12

Here is a drawing of a typical region being considered in this exercise. We are actually interested in two regions, one of which is the closed region A consisting of all points (x, y) where $a \leq x \leq b$ and $g(x) \leq y \leq f(x)$ and the other of which is the open region V consisting of all points (x, y) where $a < x < b$ and $g(x) < y < f(x)$.

Intuitively it probably seems clear that A should be the closure of V and V should be the interior of A, and that the boundaries of both regions should be the points in $A - V$. The purpose of the exercise is to justify this intuition.





(***Source:*** http://www.math24.net/definite-integral.html)

The idea is to set up a comparison with a fundamental example; namely the solid square region defined by $0 \leq x, y \leq 1$. In this case everything can be analyzed in a straightforward manner, and we generalize to regions like A and V by constructing a homomorphism from the square to A. More precisely, we construct a homeomorphism of the coordinate plane to itself which sends the solid square to A and its interior points to V (and its boundary points to $A - V$).