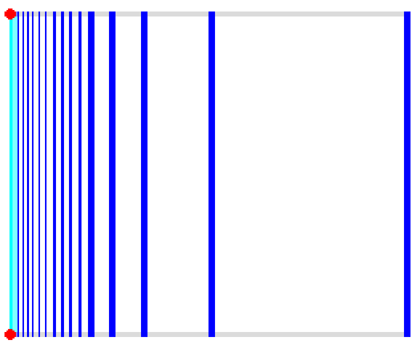
Drawing to accompany Munkres, Exercise $25.10(c)$

The following drawing of the set A should clarify many of the issues arising in the solution to this exercise. By definition, this set is given by a countable union of vertical line segments, which are depicted in blue, and two points which are depicted in red. The region shaded in light blue contains infinitely many additional closed segments which are too closely spaced to be shown due to the resolution of the drawing, but A does ***not*** contain the closed segment on the $y-$ axis which joins the two red points.



The crucial observation is that if U is a clopen subset which contains one red point, it must also contain the other. This is true because every neighborhood of a red point contains the endpoints of all but finitely many vertical segments. Since these segments are connected, the set U must contain each of these vertical segments, and since the other red point is in the closure of the union of these segments, the other red point must also be in U.

Also, here is a drawing of the set C in this exercise; the vertical and horizontal segments are in light blue, and the regions in light gray contain infinitely many segments which are too closely spaced to show. One advantage of the picture is how it shows that this set is symmetric with respect to rotation through one or more right angles. This can also be verified analytically (some additional information is given in the file `math205Asolutions03.pdf`).