Solution to Munkres, Exercise 28.2
(for finite products only)

By induction it suffices to prove this for twofold products. Suppose X and Y are locally compact Hausdorff. Then previous results imply $X \times Y$ is Hausdorff.

Let $(x,y) \in X \times Y$, and let $W \subseteq X \times Y$ be open such that $(x,y) \in W$. Take open subsets $U_0 \subseteq X$, $V_0 \subseteq Y$ such that $(x,y) \in U_0 \times V_0 \subseteq W$ (we can do this because $X \times Y$ has the product topology). Since X and Y are locally compact there are open sets $U \subseteq X$, $V \subseteq Y$ such that $x \in U \subseteq \overline{U} \subseteq U_0$ and $y \in V \subseteq \overline{V} \subseteq V_0$

$\overline{U_0}$, $\overline{V_0}$ are compact. Since $\overline{U \times V} = \overline{U} \times \overline{V}$ and the product of compact spaces is compact, it follows that

$$(x,y) \in U \times V \subseteq \overline{U} \times \overline{V} = \overline{U \times V} \subseteq U_0 \times V_0 \subseteq W$$

where $\overline{U \times V}$ is compact. ∎