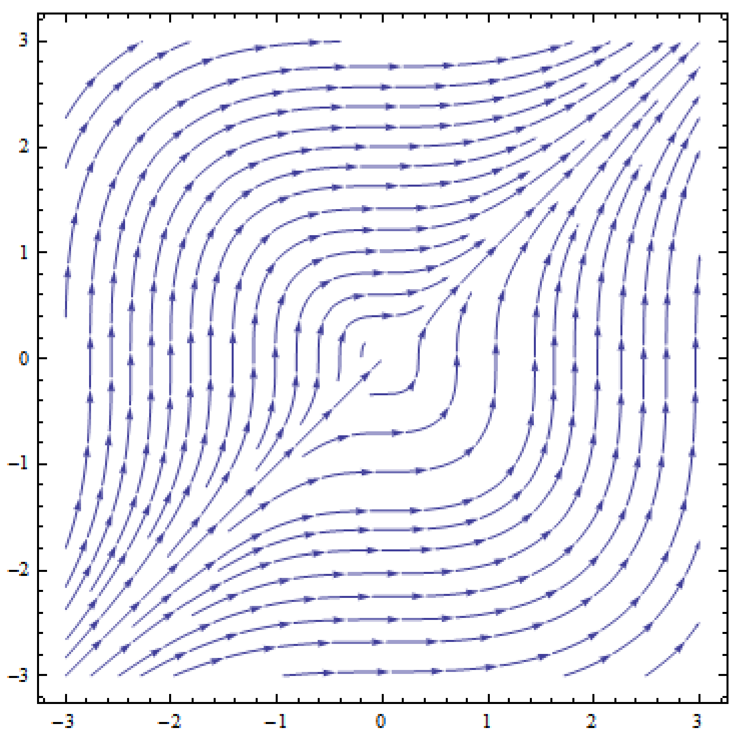
Further information on Additional Exercise A.6

We shall first say more about our assertion that the integral curves define a local action of $\mathbb{R}$ on an open set V. By definition, $\gamma(t) = \Phi(t, z)$ is the unique solution curve (or integral curve) for the system of differential equations $x' = M(x, y)$ and $y' = N(x, y)$ with initial condition $\gamma(t) = z$. Therefore the Chain Rule implies that $\beta(t) = \gamma(t + s)$ is the solution curve with initial condition $\beta(0) = \gamma(s)$. If we translate this back into a statement about the mapping Φ, we obtain the identity

$$\Phi(t, \Phi(s, z)) = \Phi(t + s, z).$$

For our example in which the local action does not extend to a global action, we have already noted that the solution curves satisfy the condition $y^3 = x^3 + C$, a fact which follows from the standard methods taught in a first course on differential equations like Mathematics **46**. A drawing which illustrates the solution curves is given below:



(***Source:*** http://math.stackexchange.com/questions/536370/how-to-show-that-a-given-vector-field-is-not-complete-in-mathbbr2)