

TOPICS FOR MATHEMATICS 205A, FALL 2014

References are to sections Munkres, *Topology* (Second Edition) (= **M**), with supplementary references to Hatcher, *Algebraic Topology* (= **H**). The outline of topics is taken from `gentopnotes2014.pdf` and `fundgp-notes.pdf`, the notes for the course. Sections from the latter which are assumed from prerequisite courses are not listed, and sections in *italics* are only covered partially or lightly; in most cases the sections cover topics probably seen in previous courses but needed as a basis for material presented here.

0. Introduction (**M** Preface, Note to the Reader)

I. Foundational material

2. *Products, relations and functions* (**M** 5–6, 8)

II. Metric and topological spaces

1. *Metrics and topologies* (**M** 12–14, 16, 20; also see `pstop-motiv.pdf`)
2. *Closed sets and limit points* (**M** 17)
3. *Continuous functions* (**M** 18–21; also see `homeomorphisms.pdf`)
4. *Cartesian products* (**M** 15, 19)

III. Spaces with special properties

1. Compact spaces (**M** 26–27)
3. *Implications of completeness* (**M** 48)
4. Connected spaces (**M** 23–25)
5. Variants of connectedness (**M** 23–25)

V. Constructions on spaces

1. Quotient spaces (**M** 22)
2. Sums and cutting and pasting (no specific reference)

VI. Spaces with additional properties

1. Second countable spaces (**M** 30)
2. Compact spaces – II (**M** 26–28)
3. *Separation axioms* (**M** 31–33, 35)
4. Local compactness and compactifications (**M** 29, 37, 38)
5. Metrization theorems (**M** 39–42)

VII. Topological deformations and approximations

0. Introduction (**H** Preface; also see `categories2014.pdf`)
1. Homotopic mappings (**M** 51–52; **H** 0–1.1)
2. *Some examples*
3. Homotopy classes of mappings (**M** 51–52, 58; **H** 0)
4. Homotopy types (**M** 58; **H** 0)

VIII. The fundamental group

0. Default hypotheses
1. Definitions and basic properties (**M** 52; **H** 1.1)
2. Important special cases (**M** 53–54, 65, 73; **H** 1.1)
3. Covering spaces (**M** 53; **H** 1.3)
4. Fundamental groups of spheres (**M** 59)
5. Simply connected spaces (**M** 53)

IX. Computing fundamental groups

1. Free groups (**M** 67–69; **H** 1.2)
2. Sums and pushouts of groups (**M** 68; **H** 1.2)
3. The Seifert – van Kampen Theorem (**M** 70; **H** 1.2)
4. Examples and computations (**M** 59, 71–72; **H** 0, 1.2)

Notes.

(1) Section I.2 is included because it contains notational conventions which are not in Munkres but are used throughout the course.

(2) Only the portions of Section III.3 before the subheading *Baire spaces* are covered in this course.

(3) The coverage of Section VI.5 will be extremely light, consisting mainly of the statements of the main results, with proofs limited to showing that metric spaces satisfy the conditions in certain definitions.

(4) Section VII.2 is included to illustrate how the concept of homotopy is related to familiar ideas involving approximating functions and small deformations of one object into another. It will be enough to know the statements of the main results; there is no need to understand the proofs.