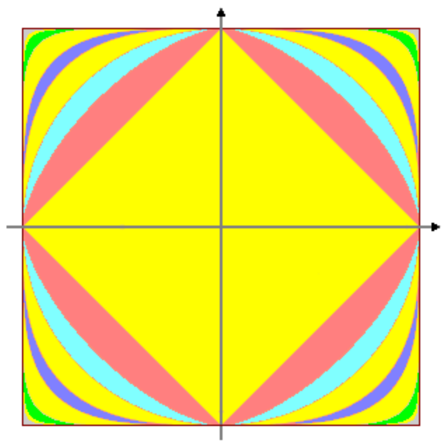
The d_p metrics on the Cartesian plane

At the end of **http://math.ucr.edu/~res/math145A-2013/product-metrics.pdf** we noted that there is a continuous family of metrics $\mathbf{d_p}$ for $\mathbb{R}^2$ which are defined for all $\mathbf{p}$ satisfying $\mathbf{1 \leq p \leq \infty}$. These metrics have the property that $\mathbf{d_q \leq d_p}$ for $\mathbf{p \leq q}$, they are continuous in $\mathbf{p}$ when $\mathbf{p}$ is finite, and $\mathbf{d_\infty}$ is the limit of $\mathbf{d_p}$ as $\mathbf{p \to \infty}$. It follows that if $\mathbf{p \leq q}$ then the unit disk with respect to the $\mathbf{d_p}$ metric is contained in the unit disk with respect to the $\mathbf{d_q}$ metric.

In the drawing below, several $\mathbf{d_p}$ unit disks in $\mathbb{R}^2$ are indicated by a range of colors. The $\mathbf{d_1}$ disk is the yellow square in the middle, the $\mathbf{d_{3/2}}$ disk is the union of the yellow square with the adjoining coral regions, the $\mathbf{d_2}$ disk is the union of the $\mathbf{d_{3/2}}$ disk with the adjoining light blue regions, and so on; the $\mathbf{d_\infty}$ disk, which is the limiting object, is the large square containing everything.



(*Adapted from* **http://www.math.ntnu.no/seminarer/perler/2004-2005.html**)

Here is a link to another picture of these unit disks:

http://yaniv.leviathanonline.com/blog/math/out-of-the-norm/

Finally, here is a link to the 3 – dimensional unit disks for the analogous $\mathbf{d_p}$ metric on $\mathbb{R}^3$:

http://www.viz.tamu.edu/faculty/ergun/research/implicitmodeling/abstracts/sm99/index.html