Addendum to Section VIII.5

Proposition Let $p : E \to B$ be a covering space projection satisfying the default hypotheses, and assume further that E is (arcwise) connected and B is simply connected. Then p is a homeomorphism.

Proof. Since p is open, continuous and onto, it suffices to show that p is 1-1.

Suppose we have $e', e'' \in E$ such that $p(e') = b = p(e'')$. Let γ be a cont. curve joining e' to e''. Then $p \circ \gamma$ is homotopically trivial because B is simply connected.* Let $h : [0,1] \times [0,1] \longrightarrow B$ be a null homotopy for the closed curve $p \circ \gamma$ in B, and let $H : [0,1] \times [0,1] \to E$ be the unique covering homotopy s.t. $p \circ H = h$ and $H(0,0) = e'$.

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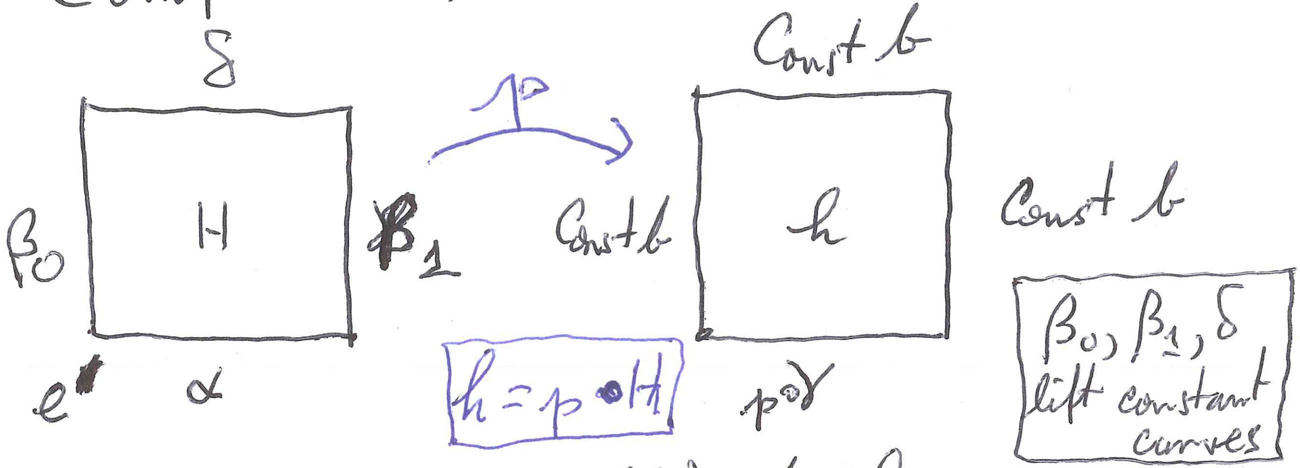
* The latter means that $\pi_1(B, b)$ is trivial for all $b \in B$.

Compare H and h:



Since $\alpha(0) = e' = \gamma(0)$, by the uniqueness of path liftings we have $\alpha = \gamma$.

Since β_0, β_1, δ map a connected set into the discrete set $p^{-1}[\{b\}]$, these maps are constant.

β_1 starts at $\alpha(1) = e'' \Rightarrow \beta_1 = \text{Const. } e''$

β_0 starts at $\alpha(0) = e' \Rightarrow \beta_0 = \text{Const. } e'$

δ starts at $e' \Rightarrow \delta = \text{Const. } e'$

δ ends at $e'' \Rightarrow \delta = \text{Const } e''$.

The last two imply that $e' = e''$. ■

It follows that simply connected spaces do not have any "interesting" coverings (assuming the default hypotheses).