The Seifert – van Kampen Theorem – I

The drawing below is meant to illustrate the first part of the proof of the Seifert – van Kampen Theorem; namely, if $X = U \cup V$ where U, V and $U \cap V$ are arcwise connected, then the images of $\pi_1(U)$ and $\pi_1(V)$ generate $\pi_1(X)$.

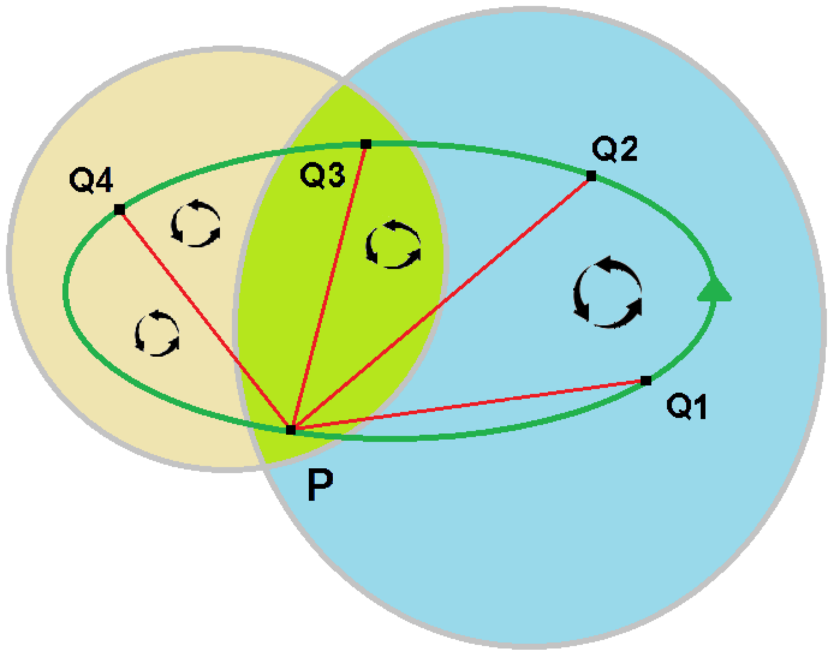

Figure 1

The idea of the proof is to split the original closed curve into finitely many arcs such that each lies in U or V; in the drawing, the endpoints of the arcs are the points **P** and $\mathbf{Q_i}$. One then joins **P** to each of the endpoints by curves which lie in U, V and $U \cap V$ depending upon which of these sets contains $\mathbf{Q_i}$. Then the original curve is basepoint preserving homotopic to the concatenation of curves of the form $\mathbf{PQ_i} + \mathbf{Q_i\,Q_{i+1}} + \mathbf{Q_{i+1}P}$. Each of these closed curves lies either in U or in V.