PREFACE

In this preface we shall give references for course materials, an extremely brief overview of the material to be covered in this course, and a few comments on some further topics which build upon the basic subject matter (but will not be needed or used in the course itself).

Course materials

The primary sources for the course will be the online notes, homework assignments and solutions to the latter which are, or will be, in the course directory:

http://math.ucr.edu/~res/math205A

Specifically, full sets of lecture notes and exercises are currently in this directory, with solutions to the exercises, drawings and further material on closely related topics. Further documents may be posted as the course progresses.

As noted in several other places, the text for the course is the following standard book on the subject:

> J. Munkres, ***Topology*** *(Second Edition).* Prentice – Hall, Upper Saddle River, NJ, 2000. **ISBN:** 0–131–81629–2.

The necessary background for the course is described at the beginning of the online notes.

Secondary course goals

All students in a general topology course have seen simplified versions of many (if not most) concepts and results from Mathematics **205A** in prerequisite courses, and indeed some of the reasons for the organization of the course go beyond the introduction of new material and the coverage of many topics in greater detail than in the prerequisites. These reasons include **(1)** the presentation of an abstract, unified approach to some basic topics in mathematics, **(2)** illustrations that increasing formality and abstraction sometimes yield simpler and more conceptual understandings of basic facts, **(3)** improvement of skills in reading and writing proofs. ***The enhancement of proof writing skills is particularly important***, and it is indispensable for studying mathematics at the graduate level. Many proofs at this level are significantly more difficult than their counterparts at the undergraduate level. In particular, the level of abstraction is usually higher, the arguments are frequently much longer or more complicated, and often the approaches are far less direct, with crucial steps sometimes relying on points which at first are easily overlooked.

Several informative articles on writing mathematical proofs can be found by searching for writing proofs and or writing proofs topology using ***Google*** or a similar search engine. General comments about such searches appear in the file on Internet resources

http://math.ucr.edu/~res/math205A/aabInternetresources.pdf which is also listed near the end of this document.

Overview of the course

The basic objectives of this course are to cover most of the basic topics *in* ***general topology***, which is essentially the same as ***point set topology***. The latter term reflects the origins of the subject in studying the properties of various subsets of the real line, and more generally of subsets of the Euclidean spaces $\mathbb{R}^n$. There are some further comments about the historical background and references to printed and online references at the beginning of the first two units in course notes, so our emphasis here will be on describing the topics that will be covered in this particular course.

A short introductory chapter (Unit **I**) gives more information on the set – theoretic background that is needed for the course. Most of this is fairly standard, so there is emphasis on some special features that are used and needed here; the most sophisticated material in the unit is the discussion of transfinite cardinal numbers in Section **3**. Unit **II** describes the abstract settings for the course; there are two levels of abstraction, the first of which is the concept of ***metric space***, which is based upon the abstract properties of distance, and the second is the concept of a ***topological space***, which is based upon the abstract properties of open regions in the real line and higher – dimensional Euclidean spaces. The basic concepts of point set topology played an important role in the mathematically rigorous development of single and multivariable calculus, so it is almost predictable that the concept of continuous function is almost as fundamental to point set topology as the abstract metric and topological spaces on which such functions can be defined. At the end of the unit we shall analyze the formal process in which one constructs the Euclidean spaces $\mathbb{R}^n$ from the real line.

In Unit **III** we concentrate on abstract versions of some crucial properties of open and closed intervals which play important roles in calculus, including the derivations of results like the Maximum Value Theorem for continuous real valued functions on closed intervals in the real line, the Intermediate Value Theorem for continuous real valued functions on arbitrary intervals, and some standard methods for finding successively better approximations to the solutions of algebraic and transcendental equations.

Although point set topology arose in connection with subsets of the real line and Euclidean spaces, the subject also provides the foundations for studying suitably defined spaces of continuous functions from one topological space to another, and this viewpoint has proven to be fundamentally important in the study of subjects like differential equations and integral equations. Some aspects of this appear in the first three units, and Unit **IV** goes further into the basic properties of these function spaces. The approach is somewhat more elementary that the one in Munkres. This material will be covered in the course if time permits, but in any case the material is worth reading because of its uses in the further study of topology and analysis.

Unit **V** describes two formal constructions which are closely related to the standard geometrical idea of constructing one object out of others by cutting and pasting. One of the topics (Section **2**) is not covered in Munkres; our reasons for including this material are that it plays important roles in the second and third courses of this sequence, and even though the details fairly straightforward it seems worthwhile to cover them by themselves rather than as parts of other discussions which involve additional concepts.

Finally, Unit **VI** discusses some more fundamental properties of subsets in the real line and Euclidean spaces, and it also considers a problem that could have been formulated right after the definitions of metric and topological spaces:

What sorts of intrinsic properties of an abstract topological space are necessary and sufficient for the topology to come from a metric?

A general treatment of this question would require more time than is available in a ten week course, so we shall limit ourselves to showing that some simple cutting and pasting constructions on metric spaces yield topological spaces that are definable by metrics (once again, to the extent that time permits). Further information on this general topic is given in Munkres and some of the supplementary files for this course. The appendices to the notes also contain information on some important additional topics that will not be covered in the course itself.

Going beyond this course

Not surprisingly, there are many different ways in which the mathematical sciences build upon the material in a general topology course, so we shall limit and organize our discussion with two basic themes:

Subsequent courses in the sequence. Topology is basically a geometrical subject, and the subsequent courses in the sequence (Mathematics **205B** and **205C**) reflect this fact, both in their basic problems and in the techniques which are employed. Ever since (at least) the beginning of the 17^{th} century, mathematicians and others have recognized the effectiveness and power of algebraic techniques for analyzing geometrical problems by transforming geometric input into algebraic terms, solving the associated algebraic questions, and translating the algebraic results back into the original geometric setting. Therefore it might not be surprising that each of the subsequent courses involves ideas and results from algebra at the graduate level.

For each of Mathematics **205B** and **205C**, the focus of the course is describable in terms of parameterized curves; the concepts of **205A** provide a means of defining such curves in arbitrary topological spaces, and the emphases of the following courses include phenomena which already appear in multivariable calculus courses.

In **205B**, the points of departure are the theorems regarding the extent to which a line integral over a curve joining two points depends, or does not depend, upon the choice of curve joining these points. In favorable situations, this leads to notions of equivalence relations such that if two curves are equivalent then their line integrals are equal (more precisely, the equivalence relations themselves are defined generally, but the results on independence of path require special types of examples). The subject matter of **205B** also reflects an important aspect of topology that receives minimal attention in **205A**; namely, its popular image as a "rubber sheet geometry" in which two objects are topologically equivalent if one can be obtained from the other by an elastic deformation (for example, bending or stretching or compression). Finally, the methods and results in **205B** have many strong ties to abstract group theory, and particularly to the theory of infinite groups.

One way to motivate **205C** is to say that it describes some extra structure on topological spaces which suffices to define a viable concept of smoothly parameterized curves. A parallel motivation is that it provides an abstract setting for studying questions about curves and surfaces which arise in classical differential geometry. In this course, there are also crucial ties to linear algebra and its generalizations, abstract multivariable calculus, and fundamental results on the theory of solutions to ordinary differential equations which go beyond the standard lower level undergraduate courses.

Further topics in general topology. Topological spaces arise naturally in nearly every branch of the mathematical sciences, and in order to keep the discussion within reasonable bounds we shall concentrate on some areas outside of topology and geometry where the impact of general topology is most pronounced. A few online references are given below. The first two involve D. Rusin's wide ranging online overview of present day mathematics, and the third is the comprehensive site at York University (in Canada, located near Toronto) devoted to general topology.

http://www.math.niu.edu/~rusin/known-math/index/54-XX.html

http://www.math-atlas.org/

http://at.yorku.ca/topology/

Further methods and results in general topology play a crucial role in ***functional analysis***, which is an abstract study of large spaces of functions largely aimed at studying questions like solutions to (ordinary and partial) differential equations and good approximations to functions. Another branch of analysis in which topological spaces and their refinements play a key role is ***measure theory***. Topological spaces also figure importantly in some foundational areas. One example is the subject between general topology and mathematical logic which is called ***set – theoretic topology***, and another is the appearance of topological spaces in certain issues from ***theoretical computer science***. Further information on the latter is included near the end of Section **VI.3** in the course notes. For the sake of convenience, here are online references for the other topics mentioned in this paragraph:

http://en.wikipedia.org/wiki/Functional_analysis

http://en.wikipedia.org/wiki/Measure_theory

http://en.wikipedia.org/wiki/Set-theoretic_topology

Some general comments on the use of ***Wikipedia*** articles as authoritative references appear in the online file

http://math.ucr.edu/~res/math205A/aabInternetresources.pdf

and further references for (generally very reliable) information on mathematical topics through the level of current research are given below:

http://mathworld.wolfram.com

http://en.wikipedia.org/wiki/Main_Page

http://planetmath.org

Final remark. There are also several sites on the World Wide Web which describe other courses covering roughly the same material as this one; we shall only list one which contains links to solutions for many of the exercises in Munkres.

http://www.math.ku.dk/~moller/e03/3gt/3gt.html