Math 205A
Fall 2003
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Final REMARKS

December, 2003

You may think this is the end ... but ... it is ... the beginning

[*cf.* the end of the David Lee Roth video for "California Girls"]

Mathematics 205A leads directly to the remaining two courses in the introdctory graduate geometry/topology sequence and also to the qulifying examination for the 205 sequence. For these reasons the written course materials will remain available in the course directory directory `~res/math205A` and the Web-accessible site

`http://www.math.ucr.edu/~res/math205A`

with subsequent additions to the latter that might be helpful. Since the main file of notes for the course is fairly long, I have added PostScript and PDF files for the various course units as well as the supplementary material. The file names for the former have the form `unit`n`.*` where n ranges between 1 and 5, and the file names for the latter have the form `addenda.*` where $*$ = either `ps` or `pdf`.

I also plan to add files of the form `qexamprep.*` that may be helpful in preparing for the qualifying examination. This document will include some remarks on further references and information on the topics that students are expected to know when taking the examination. Two particular reasons for posting such a document are

(1) the material tested on the qualifying examinations is not necessarily identical to the material covered in the course although the difference between them is supposed to be small,

(2) the current departmental outlines for Mathematics 205ABC are hopelessly obsolete.

The material above and a summary of files added during the past two weeks may be found in the documents `finalremarks205A.*` in the directories mentioned above.

Thus far I have added the following files to the course directory on the Department network and the Web-accessible directory cited above; as usual, the asterisk indicates that the files are available in `dvi`, `ps` and `pdf` formats.

`ghostlink` (*This is just a text file.*)
This file gives the link from which Ghostview software for reading PostScript files can be obtained. As noted in other course materials, the Acrobat PDF files sometimes have defects that are not present in the PostScript files.

`gradescaling.*` (REVISED AND UPDATED VERSION)
This file contains complete information on the basis for computing course grades.

`exam2a.*`
These files contain the solutions for the problems on the second course examination.

`Lambertfcn.*`
These files provide additional information on the inverse to the function $f(x) = x + e^x$ which figures in the solution to Additional Exercises 0 for Section IV.3 of the course notes. This is related to the W-function considered by Lambert in the eighteenth century. If I am able to create or find a reference for a rigorous proof that this inverse function is not a standard function from first year calculus, I shall do so, if for no other reason than to satisfy my own curiosity. Although it seems extremely likely that one cannot write the W-function in this fashion, a web site written by a reputable mathematician claimed that no one had yet given a rigorous proof of this and added that he and a coworker were trying to do so.

`more2countable.*`
In Section VI.1 of the course notes I claimed that there are no logical relations among the concepts of second countability, separability and the Lindelöf Property for nonmetrizable topological spaces except that second countability implies the other two properties. These files give examples to show that

(a) separability does not imply the Lindelöf property (hence also does not imply second countability),

(b) the Lindelöf property does not imply separability (hence also does not imply second countability),

(c) separability and the Lindelöf property together do not imply second countability.

These examples are different from the ones presented in Munkres and do not involve order topologies. Tychonoff's Theorem on products of compact spaces and some material from the remainder of Unit VI are needed and used in these documents.

Since the three concepts are equivalent for metrizable spaces, none of these examples can be metrizable, but the examples in (b) and (c) turn out to be $\mathbf{T_4}$ while the example in (a) is $\mathbf{T_3}$. Verification that that the examples in (b) and (c) are $\mathbf{T_4}$ is left to the reader as an exercise. It is also left to the reader to think about whether a separable $\mathbf{T_4}$ space has the Lindelöf Property.