

Mathematics 205A

Course Notes

Fall 2003

Department of Mathematics

University of California, Riverside

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