

**Mathematics 205A**

**Topology — I**

**Course Notes**

**Revised, Fall 2005**

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## Comments on the 2005 version

September, 2005

These notes are very close to those for my Fall 2003 course, but there are a few differences.

- (1) There have been some minor changes in the first unit and inclusion of online references to a course in set theory (Mathematics 144) that I am also offering in Fall 2005.
- (2) Some material has been added to the section on second countable spaces to describe examples of spaces that are separable and/or Lindelöf but not second countable.
- (3) The most important change is that Unit IV has been replaced by entirely different material on the topological properties of spaces of continuous functions. The treatment differs significantly from that in Munkres; these notes only consider certain special cases that are adequate for many applications and for which one can give greatly simplified proofs.
- (4) There is now only one required text. References to the other have been removed from the body of the notes.
- (5) There have been some minor alterations to the listing of materials in the course directory.