Math 205A
Fall 2005
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UPDATED GENERAL INFORMATION — OCTOBER 12, 2005

Here is a clarification for the proof that $A \cup \mathbf{L}(A)$ is closed on page 18.

As noted in the argument, if $y \notin A \cup (A)$, then there is an open neighborhood U_y of A such that $U_y \cap A$ is empty. This immediately implies that $U_y 7$ lies in the complement of A, but it **also** implies that U_y lies in the complement of $\mathbf{L}(A)$, for if $z \in U_y$ then U_y is an open subset containing z such that $(U_y - \{z\}) \cap A$ is empty. Thus we have $U_y \subset X - \overline{A}$ for all y, and in the next to last line of the proof we can replace the $X - A$ on the extreme right hand side of the set-theoretic inequality by $X - \overline{A}$. This provides the information needed to justify the final line of the proof and hence to show that $\overline{A}$ is closed.■

Since we are commenting on the notes, we shall also insert the proof that if $A \subset B \subset X$ then $\mathbf{L}(A) \subset \mathbf{L}(B)$. — If $y \in \mathbf{L}(A)$, then for every open set U containing y we have $(U - \{y\}) \cap A$ is nonempty. On the other hand if $A \subset B$, then

$$(U - \{y\}) \cap A \quad \subset \quad (U - \{y\}) \cap B$$

and since the first of these is nonempty so is the second. Therefore we must also have $y \in \mathbf{L}(B)$.■