

## A quotient map which is neither open nor closed

The example in Exercise 3 on page 145 of Munkres is given by taking  $A = \mathbb{R} \times \{0\} \cup [0, \infty] \times \mathbb{R}$  and letting  $f : A \rightarrow \mathbb{R}$  send  $(x, y) \in A$  to  $x$ . It is not closed, for if  $B$  is the hyperbola defined by the equation  $xy = 1$  and  $C = A \cap B$  then  $C$  is closed in  $A$  but its image under  $f$  is the nonclosed set  $(0, \infty)$ . It is not open, for if  $W \subset \mathbb{R}^2$  is the open rectangular region  $(-2, 2) \times (1, 2)$ , then  $V = W \cap A$  is open in  $A$  but its image under  $f$  is the nonopen subset  $[0, 2)$ .

There are several ways to check that  $f$  is a quotient map. The quickest is to use the preceding exercise; by the latter, if we can find a map  $\sigma : \mathbb{R} \rightarrow A$  such that  $f \circ \sigma$  is the identity, then  $f$  is a quotient map. If we take  $\sigma(x) = (x, 0)$ , then  $\sigma$  has the required property. ■