Math 205B
Winter 2014

UPDATED GENERAL INFORMATION — JANUARY 30, 2014

Review for the first examination

The examination will cover Sections I.0–I.3, II.1 and III.1–III.4 in `algtop-notes.pdf` (along with the corresponding sections in Munkres and Hatcher listed at the beginning of these section). However, the Addendum to Section III.1 will not be covered. Also, the material on computing fundamental groups using the Seifert-van Kampen Theorem will not be covered.

Files of exercises. The file `exercises00-2012.pdf` contains background and review questions, and this material will not be covered (although knowledge of it is assumed, and working the problems might be worthwhile). In contrast, the contents of `exercises01-2012.pdf` are part of the material covered; two misprints which cause trouble with problems 3 and 5 have been corrected. As noted in the Wednesday lecture, the material in exercises 1–3 and 11–12 in `exercises02-2012.pdf` will not be covered. I plan to write out solutions for exercises from the latter two sets; if a solution to a problem is not given, that means that there is no need to be concerned about solving that particular problem.

Old examinations on file. Examinations from the Winter 2012 version of this course are in the course directory, and the folder `old-exams` in this directory also contains earlier examinations (in class and take home) which may be useful for review.

Priorities for knowing and understanding. The statements of the main results from Unit I (the lifting criterion, the description of the group of covering transformations $\Gamma(E \to B)$, and the classification of covering spaces) should be known well enough so that these can be applied to specific types of examples like those in some of the posted problems from earlier exams. Also, the examples from Section II.1 for realizing cyclic groups as fundamental groups should be known. For Section III.1, both the statements and proofs of all results through Proposition 6 should be known, and likewise for Propositions 1 and 2 from Section III.2; it will be enough to know how to outline the proofs of other results from these sections (but as noted above Theorem III.1.8 is not included in the exam material). In Section III.3, the statements of results should be known, and likewise for the proofs beginning with that of Theorem 3, and in Section III.4 all the statements and proofs should be known. For Sections III.3–III.4, it is likely that there will be applications of the results to specific graphs (finding fundamental groups and maximal trees, or finding fundamental groups of n-sheeted coverings for some or all values of n), so it is worthwhile to look at a variety of moderately complicated examples like those from previous examinations. One such example is given in the file `graphpix5.pdf`.

Finally, here are a couple of problems that were considered but not included on the exam. In each case, there is a default hypothesis that all spaces which arise are Hausdorff and locally arcwise connected.

1. Suppose that $E \to B$ is a covering with $\pi_1(B)$ finite of order 2^n and E connected. Prove that the number of sheets in the covering is a power of 2.

2. Let $(X, \mathcal{E})$ be a connected graph with finitely many edges, and let a, b, c be distinct vertices. If Γ_1 is a reduced edge path joining a to b and Γ_2 is a reduced edge path joining b to

c, is the concatenation $\Gamma_1 + \Gamma_2$ a reduced edge path joining a to c? Either prove this or give a counterexample.

3. Suppose that the connected graph $(X, \mathcal{E})$ has n vertices. Find a sharp lower bound for the Euler characteristic $\chi(X)$; in other words, find an integer B such that $\chi(X) \geq B$ always holds and there is some connected graph for which one has equality. You should get an answer of the form $\frac{1}{2}\,p(n)$ where p is a polynomial with integral coefficients that are given explicitly.

4. Given a continuous mapping $f : X \to Y$, the mapping $\sigma : Y \to X$ is said to be a *cross section* of f if the composite $f \circ \sigma$ is the identity on Y. — Suppose that $p : E \to B$ is a covering with E connected and p has a cross section. Prove that p is a 1-sheeted covering space.