

Mathematics 205B, Winter 2008, Take-Home Assignment

This will be due on **Monday, March 10, 2008, at 1:00 P.M.** either earlier that day in class, in my mailbox, or at the front desk of Surge 202. If you wish to use some version of \TeX in writing up your answers, please feel free to do so. *You must show the work behind or reasons for your answers.*

1. Prove one of the two assertions involving strong deformation retracts which appear on page 51 of the commentaries.
2. Let X be a lens space S^{2n+1}/\mathbf{Z}_k where $k > 1$ is an odd integer. Prove that X does not have a connected covering space with an even number of sheets.
3. Suppose that we are given a covering space projection $p : E \rightarrow B$, where E and B are Hausdorff, connected, and locally arcwise connected, and suppose that p is homotopic to a constant map. Prove that E is simply connected. Give an example of a covering space projection which has this property such that B is not contractible.
4. Suppose that W is an open subset of \mathbf{R}^n which is a union of two open connected subsets U and V such that $U \cap V$ is simply connected and $\pi_1(W)$ is abelian. Prove that either U or V is simply connected. [*Hint:* Translate this into a statement about free products of groups.]