Comments on the practice problems

1. Every point in U has a neighborhood base of convex open subsets.

2. [corrected] Display the diagram

$$\begin{tikzcd} \pi_1(U_1 \cap U_2) \arrow[r, "\text{onto}"] \arrow[d, "\text{trivial}"'] & \pi_1(U_1) \arrow[d] \\ \pi_1(U_2) \arrow[r] & \pi_1(X) \end{tikzcd}$$

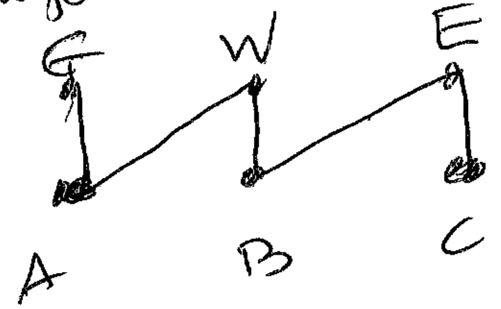
Now $\pi_1(X)$ is generated by the images of $\pi_1(U_1)$ and $\pi_2(U_2)$. If $a \in \pi_1(U_1)$ then its image comes from $\pi_1(U_1 \cap U_2) \to \pi_1(U_1) \to X$ which equals $\pi_1(U_1 \cap U_2) \to \pi_1(U_2) \to \pi_1(X)$ and the latter is trivial. Hence the image of $\pi_1(U_2)$ generates $\pi_1(X)$, so that the latter $\cong$ quotient of the former.

3. There are nine edges:

AG	AW	AE
BG	BW	BE
CG	CW	CE

Hence the Euler characteristic equals $6-9=-3$ and the fundamental group is free on $1-(-3)=4$ generators.

A maximal tree has #vertices $-1=5$ edges + contains all vertices. Here is the simplest (but not unique) example.



4. The lifting criterion applies to show that $j: A \to B$ lifts to $J: A \to E$. We claim J maps A homeomorphically to its image. Since A is compact and E is T_2, need only show J is 1-1. But $J(u)=J(v) \Rightarrow pJ(u)=pJ(v) \Rightarrow j(u)=j(v)$ $\Rightarrow u=v$ since j is 1-1.

The map $p \mid J[A]$ is an inverse to the induced homeo $J' : A \to J[A]$.

5. (a) π_1 is free on $1 - \chi = 1 - (V - E) = E - V + 1$ generators, so the latter is nonnegative and hence $E + 1 \geq V$. If X is a tree we have equality since $\chi = 1$. If not, then $1 - \chi > 0$, so there is strict inequality.

(b) Consider all pairs (v, e) where v is a vertex of e. Since each e has two vertices there are $2E$ such pairs. Also, there are kV_k such pairs where the vertex lies on k edges. Summing these yields $2E = \sum kV_k$.

6. Let E_1, V_1 and E_2, V_2 be the numbers of ~~vertices~~ edges & vertices for X_1 & X_2 respectively. Then $V_1 - E_1 = 1 = V_2 - E_2$. Now X has $E_1 + E_2$ edges and $V_1 + V_2 - 1$ vertices, so $\chi(X) = V_1 + V_2 - 1 - (E_1 + E_2) = (V_1 - E_1) + (V_2 - E_2) - 1 = 1 + 1 - 1 = 1$. Hence X is a tree.