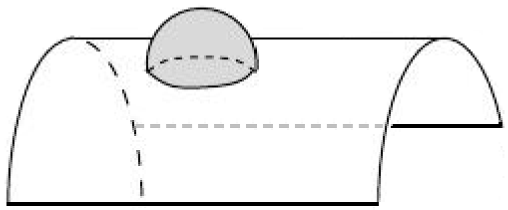
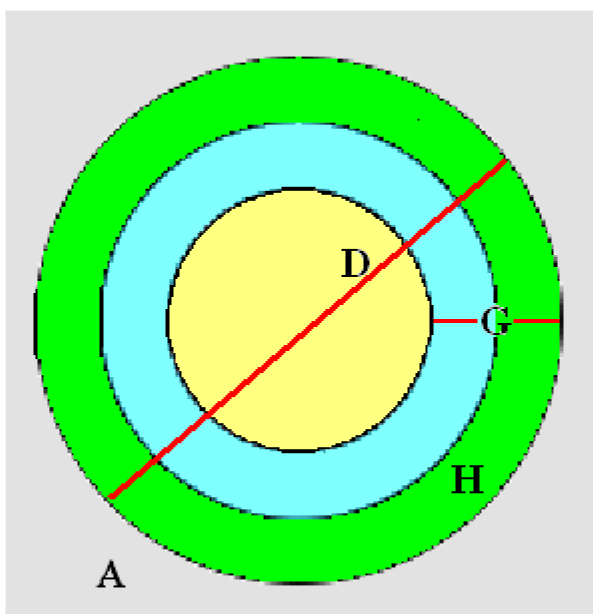
REGULARLY ATTACHING A CELL TO A SPACE

A space X is obtained from a subspace A by regularly attaching a k – dimensional cell if X is the union of A and a closed subspace D which is homeomorphic to a k – disk, such that the intersection of A and D corresponds to the boundary sphere of the disk. In the drawing below, A is given by the cylindrical surface, and the 2 – cell is the "bubble" shaded in gray. The intersection is just the circle on the boundary of the 2 – cell.



It is intuitively clear that every 2 – dimensional polyhedron in 3 – space is obtained by a finite sequence regular cell attachments with cells of dimensions 0, 1 and 2, and as indicated in the notes it is possible to prove this rigorously.

One important fact about cell attachment is that the inclusion map of pairs from (D, S) to (X, A) induces isomorphisms in homology. In the course of that proof one introduces subsets G and H of D, where G corresponds to the closed annulus of points whose distance from the center is between ½ and 1, and H corresponds to the open annulus of points whose distance from the center is greater than ¾ (and less than or equal to 1). All these subspaces are depicted in the drawing below:



The notion of regular cell attachment is formally introduced on page **534** of Hatcher.