The Möbius Strip

One way to define the Möbius strip is to take
$M = [0,1] \times [-\frac{1}{3}, \frac{1}{3}] / R$ where R is the equivalence relation whose equivalence classes with more than one point are $\{(1,s), (0,-s)\}$

We can realize this space as a subspace of $\mathbb{R}^3$ explicitly by the following map, given by cylindrical coordinates (r, θ, z)

$$r(t,s) = 1 + s\cos \pi t$$

$$\theta(t,s) = 2\pi t \qquad z(t,s) = s \sin \pi t.$$

One can check directly that (t,s) and (t',s') map to the same point of $\mathbb{R}^3 \Leftrightarrow$ $(t,s) = (t',s')$ or one of (t,s) & (t',s') is $(0,s)$ and the other is $(1,-s)$. This yields a well-defined continuous 1-1 map $M \to \mathbb{R}^3$, which must be a homeomorphism onto its image.