Cantor and collections of objects too large to be sets

The following discussion, which is taken from an undergraduate set theory textbook, summarizes the standard mathematical viewpoint on the difference between sets and arbitrary collections of objects. It begins with a quotation of Cantor.

A set is a Many that allows itself to be thought of as a One.

Georg Cantor

The mathematical community [soon realized] the concept of "set" could not be applied carelessly to any collection that could be comprehended by the human mind. To use Cantor's language, some groups of *many* do not allow themselves to be thought of as a single *one*. It became apparent that some collections could safely be called 'sets' ... and some ... were not to be considered sets. How then was the mathematical community to tell which were safe collections and which would lead to contradiction? The answer lay in carefully considered axioms that would give criteria for identifying collections that are sets.

Source:

C. Schumacher, Chapter Zero: Fundamental Notions of Abstract Mathematics (Second Edition). Addison-Wesley, Boston *etc.*, 2001, pp. 39, 52-53.

Self-referencing statements

Many of the apparent paradoxes in set theory which were discovered during the late 19th and early 20th centuries involved self-referencing statements like "Never make sweeping generalizations." There is an extensive discussion of such statements, both in and out of mathematics, in the following article from the online <u>Stanford Encyclopedia of Philosophy</u>. Here is a link:

https://plato.stanford.edu/entries/self-reference